

Counting in binary: The 16 Card Trick¹

The Trick (basic)

1. The participant chooses a random card from a deck of 16, memorises it, replaces it, and shuffles the deck as much as desired.
2. The dealer creates two face-up piles of eight cards, dealing the top card onto the left, then right deck and repeating until all the cards are gone.
3. The participant then points to which pile contains the chosen card.
4. The dealer then re-stacks the piles into a new deck putting the pile that contains the participant's card in the first round of steps 2-4 at the **bottom** (at next repeat of 2-4 on **top**, then at the **bottom** and at last repeat on **top**)
5. Steps 2-4 are repeated three times, done 4 times in total (re-stack pile with chosen card as bottom, top, bottom, and finally top).
6. The dealer then flips over 5 cards from the top and reveals the 6th card to be the original chosen card.

The Trick (advanced)

1. The participant chooses a random card from a deck of 16, memorises it, replaces it, and shuffles the deck as much as desired. The participant also states a random number between 1 and 16 (call this number n).
2. The dealer creates two face-up piles of 8 cards, dealing the top card onto the left, then right deck and repeating until all the cards are gone.
3. The participant then points to which pile contains the chosen card.
4. The dealer then re-stacks the piles into a new deck. The dealer must maintain the order of the specific piles themselves however.
5. Steps 2-4 are repeated three times.
6. The dealer then flips over $n-1$ cards from the top and reveals the n th card to be the original chosen card.

For the advanced version of the 16 card trick you need to **count in binary** (see overleaf). Suppose $n=6$ as in the basic version. Then write $n-1=5$ in binary which is $101=0101$ (you can add as many zeros to the left as you want). Reading 0101 from the right tells you how to re-stack (1=bottom, 0=top). So, you re-stack the pile with chosen card at bottom, then top, then bottom, then top.

¹ The 27 card trick is based on ternary numbers.

Counting in binary

In our usual way of writing numbers (the **decimal system**) we use 10 digits:

0,1,2,...,9.

In the **binary system** we use 2 digits: **0** and **1** (“bi” means “two” as in bicycle, binoculars...)

In our usual (**decimal system**) we count one, two, three, ..., nine and write 0,1,2,...,9. Having used up all digits the next numbers ten, eleven, twelve... nineteen are written as 10, 11, 12, 13, ..., 19. We may write zeros to the left without changing the meaning of the numbers. So, decimal counting goes as

000, 001, 002, ... 007, 008, 009, (rightmost digit is reset to zero, and the digit to its left is incremented)

010, 011, 012, ...

...

090, 091, 092, ... 097, 098, 099, (rightmost two digits are reset to zeroes, and next digit is incremented)

100, 101, 102, ...

In the **binary system** we count similarly as follows (using only digits 0 and 1):

0000,

0001, (rightmost digit starts over, and next digit is incremented)

0010, 0011, (rightmost two digits start over, and next digit is incremented)

0100, 0101, 0110, 0111, (rightmost three digits start over, and the next digit is incremented)

1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111 ...

decimal	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

For instance, the decimal 5 is 101 in binary, and 11 in decimal is 1011 in binary.

What is the usual (decimal) 21 in binary?