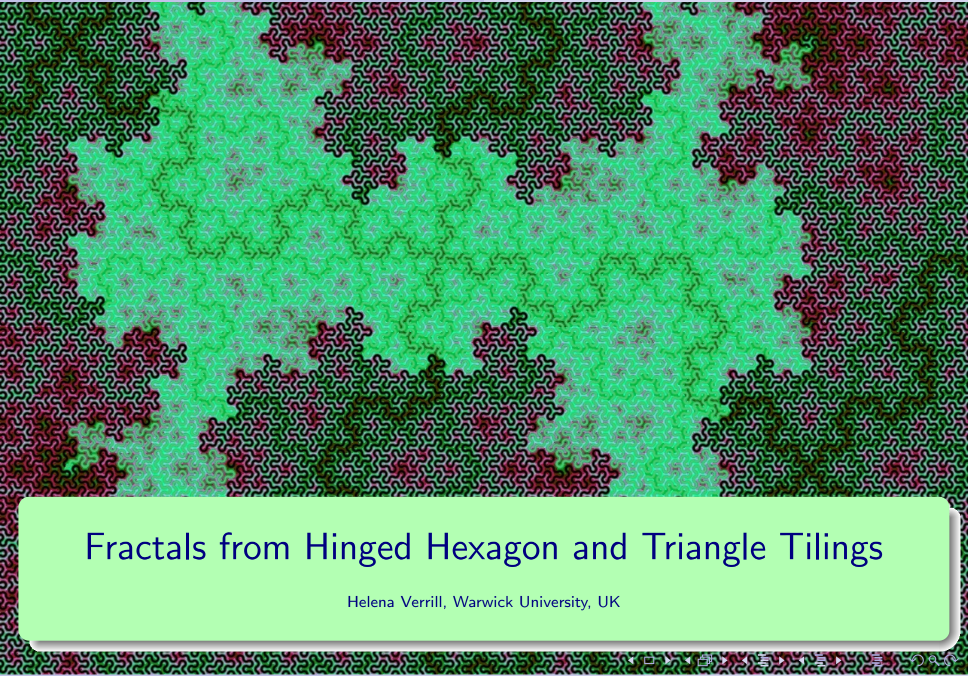
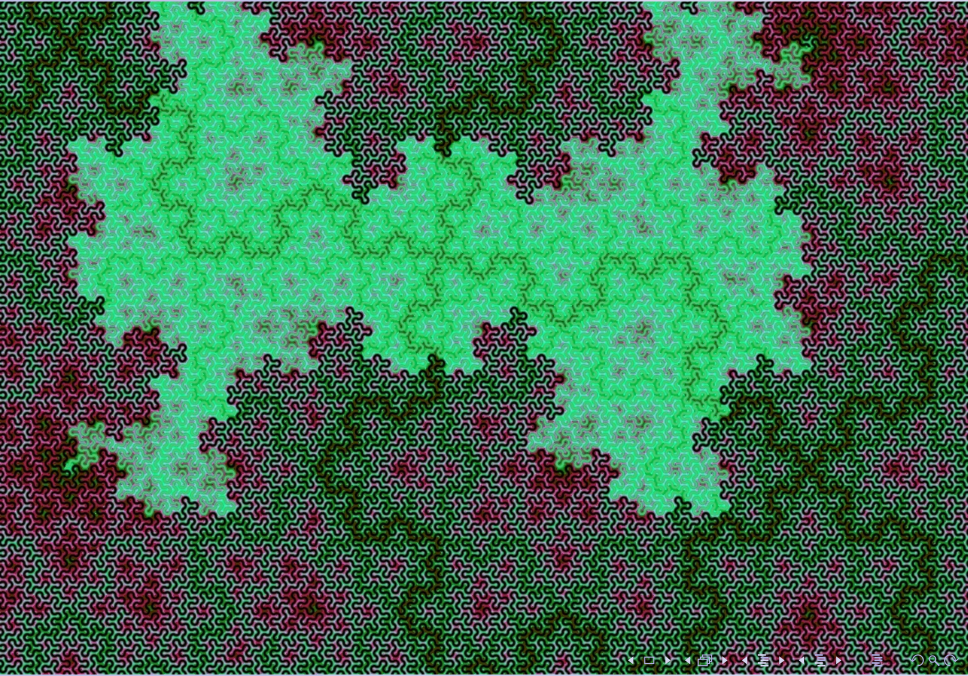


Fractals from Hinged Hexagon and Triangle Tilings

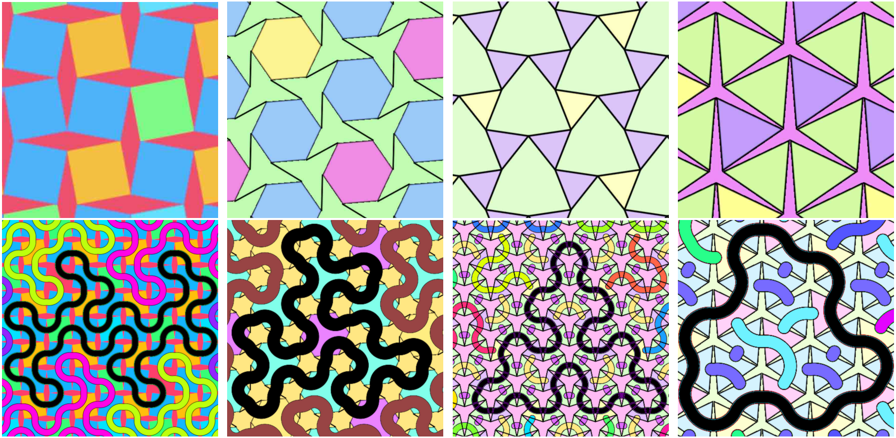
Helena Verrill, Warwick University, UK



Summary of Talk

(Graphics from `https://www.mathamaze.co.uk/Truchet2/`)

Take an initial tiling, e.g., of squares, hexagons or triangles.



Summary of Talk

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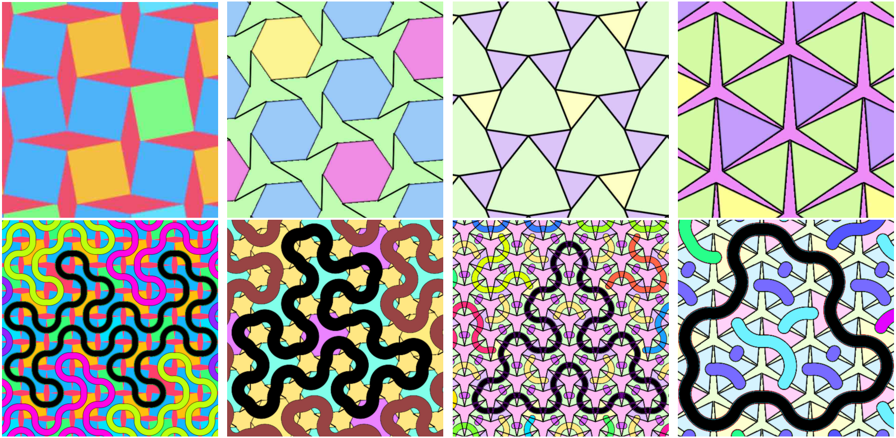
Decorate each tile with arcs;



Summary of Talk

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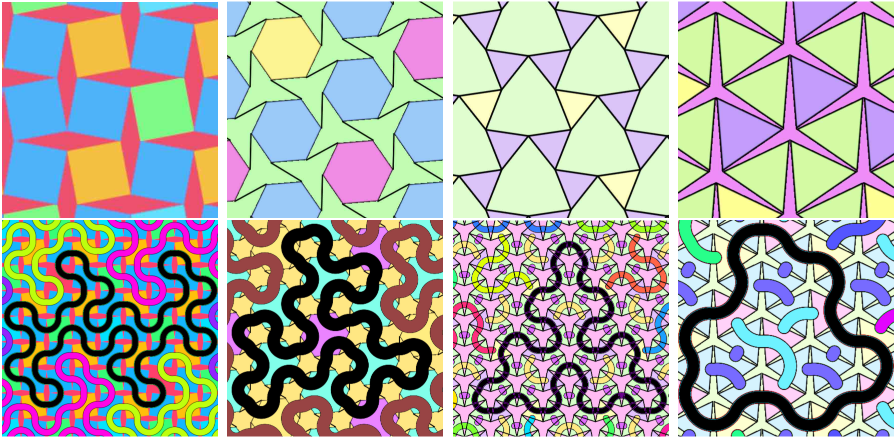
Decorate each tile with arcs; hinge to obtain new tiling;



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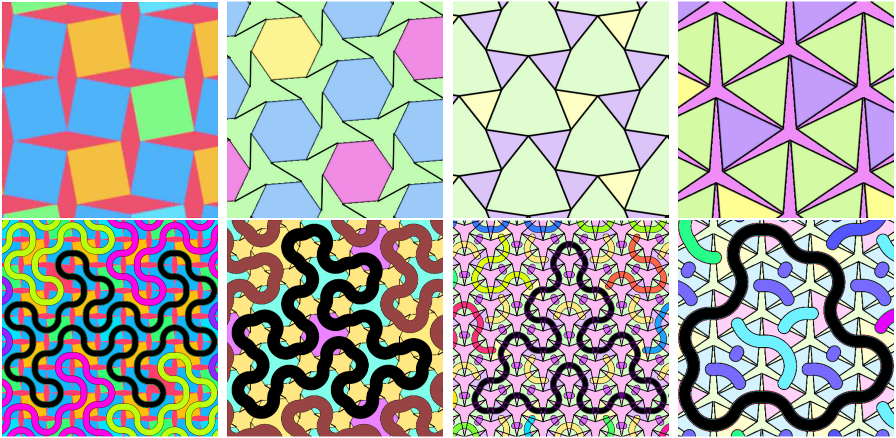

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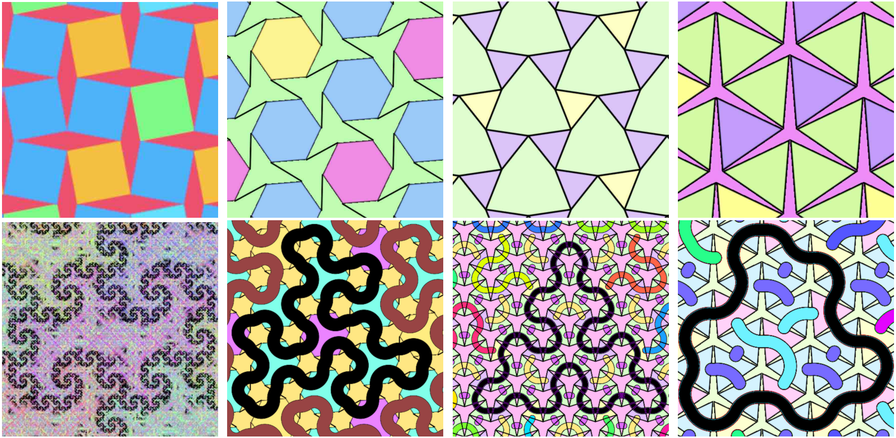

Heighway dragon

Summary of Talk

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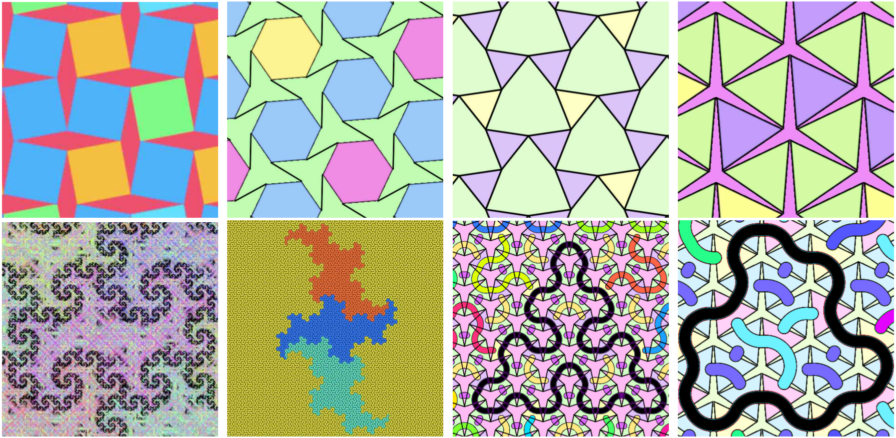

Heighway dragon • Davis/Knuth terdragon

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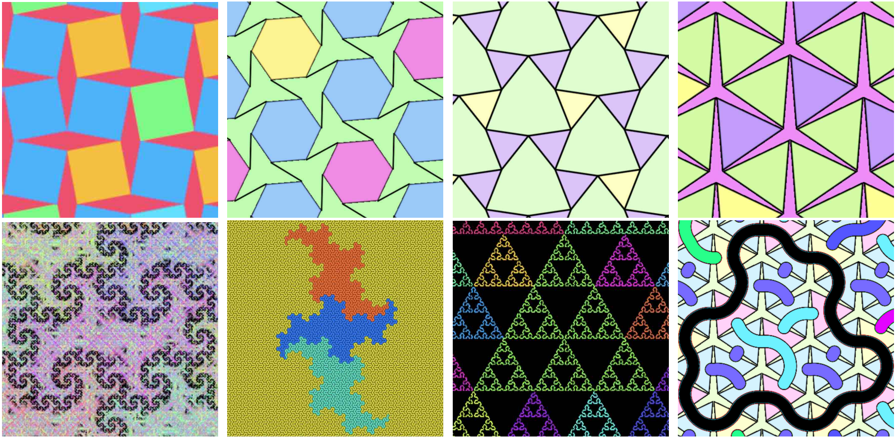

Heighway dragon • Davis/Knuth terdragon • Sierpinski triangle

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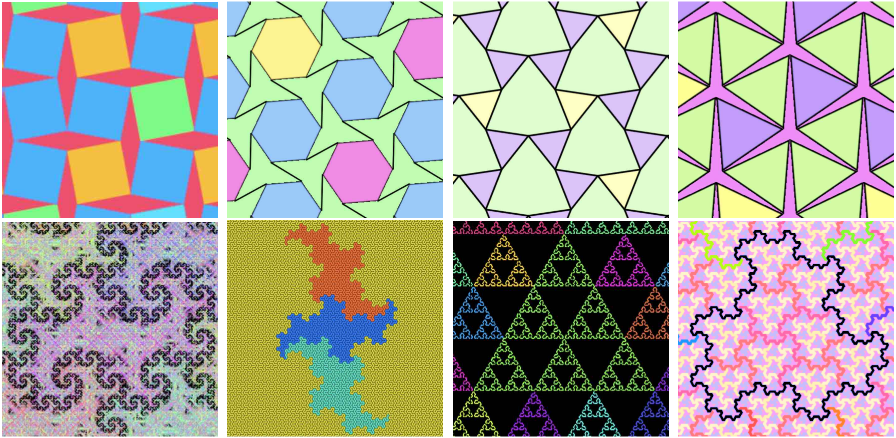

Heighway dragon • Davis/Knuth terdragon • Sierpinski triangle • Terdragon b'dary

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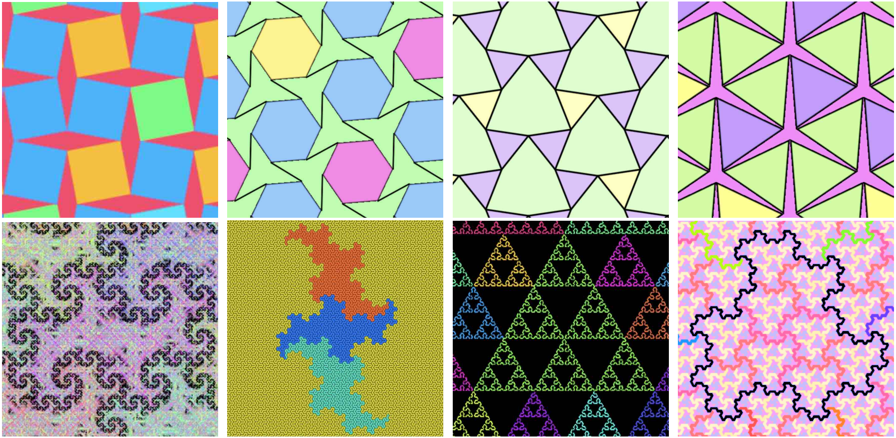
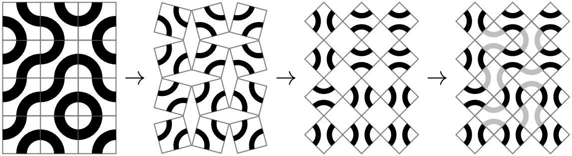


Heighway dragon • Davis/Knuth terdragon • Sierpinski triangle • Fudgeflake

Recall from last year: Hinged squares

Hinged
Truchet tiling



Hinged tiling;
rotate 45°;
scale by $\sqrt{2}$;
background
becomes
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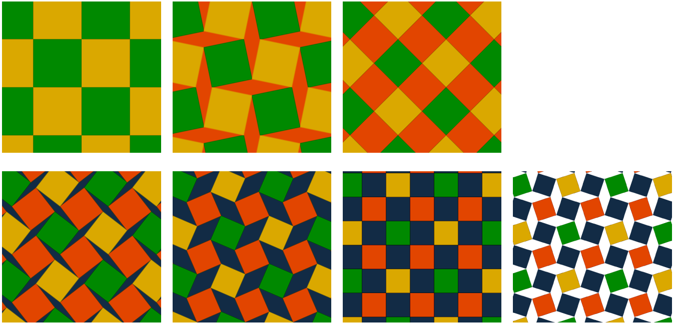

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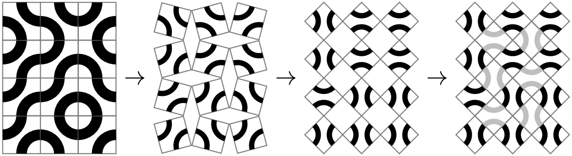

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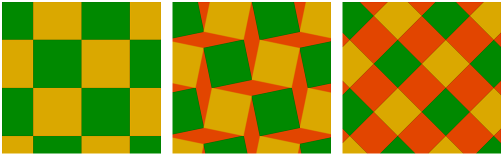
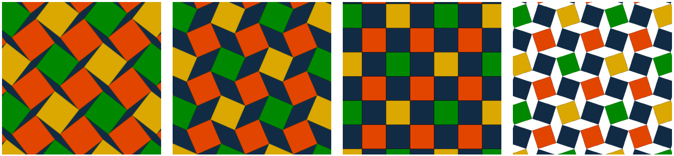


End result:
Heighway
fractal dragon



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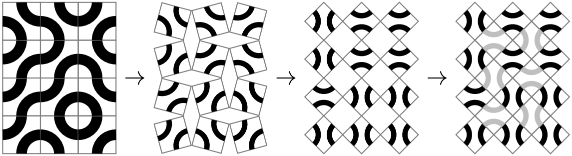

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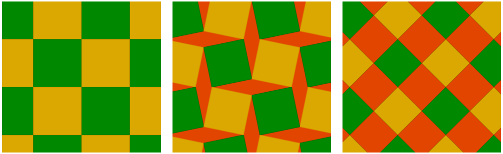
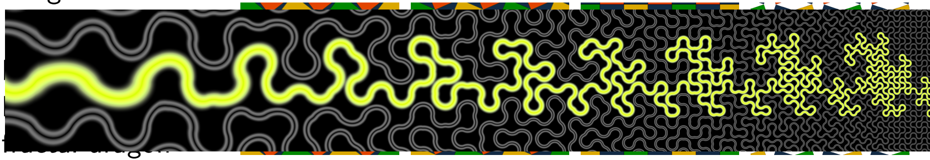
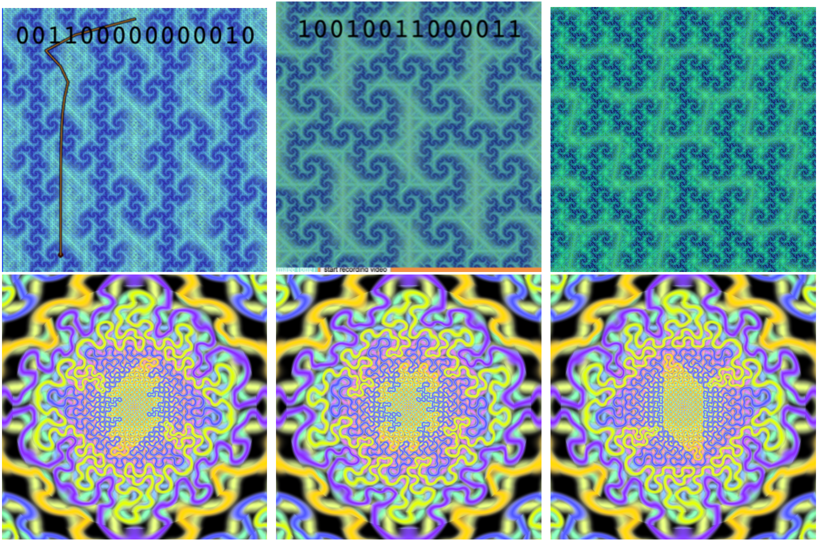




(this is from last year's talk; spot Heighway's dragon)



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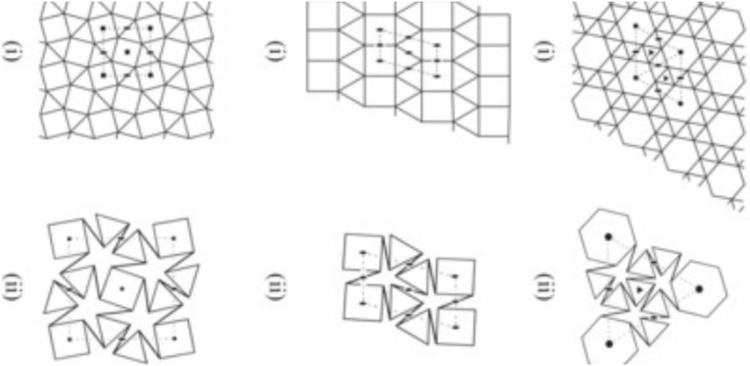

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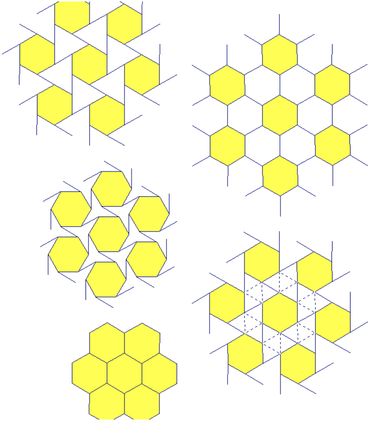

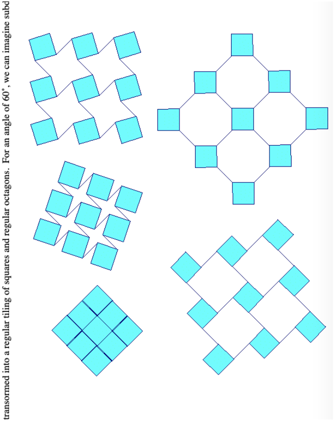

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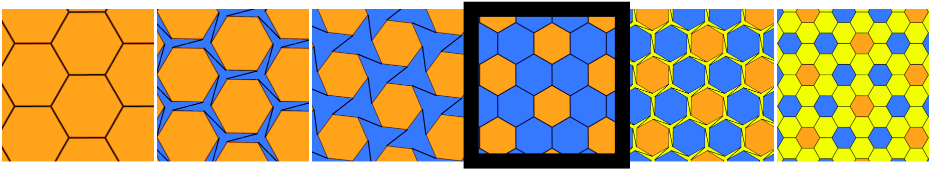
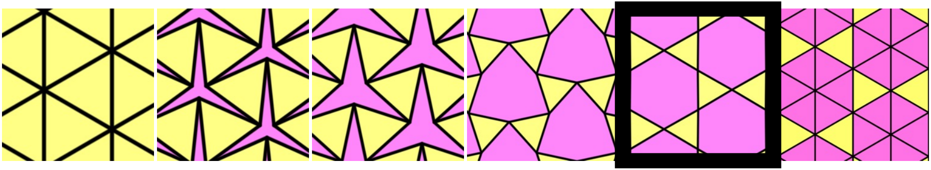
- The following slides contain snapshots from javscript programs, which are used to clarify ideas. Please follow along at: `www.mathamaze.co.uk/Truchet3/`

The hinged tilings

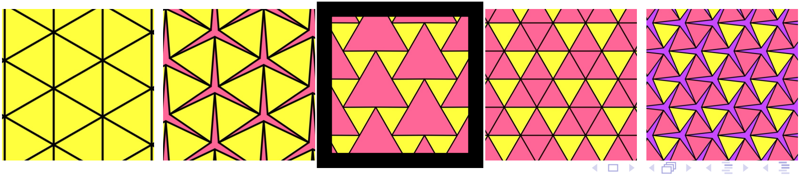
Hexagons with links: (rotate hexagons through 30°)



Triangles (1) (rotate triangles through 60°)



Triangles (2) (rotate triangles through 30°)



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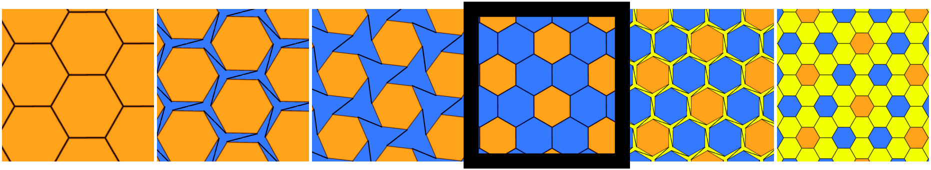

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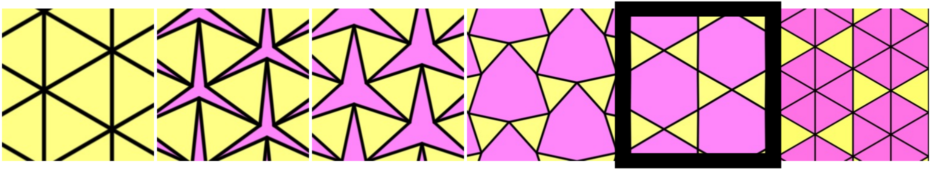

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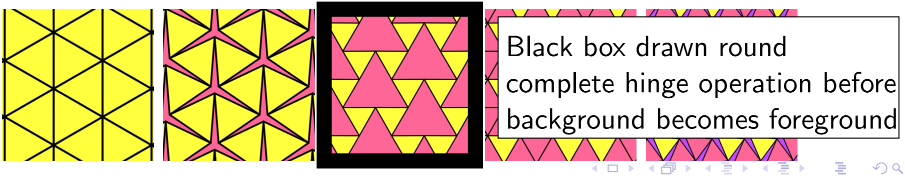

Black box drawn round complete hinge operation before background becomes foreground

Truchet designs and fractals

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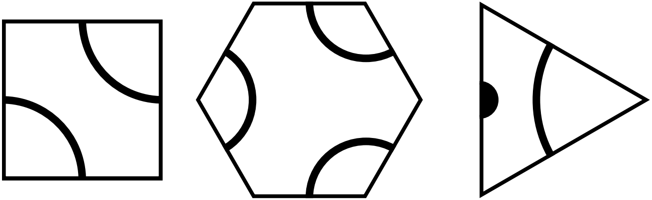

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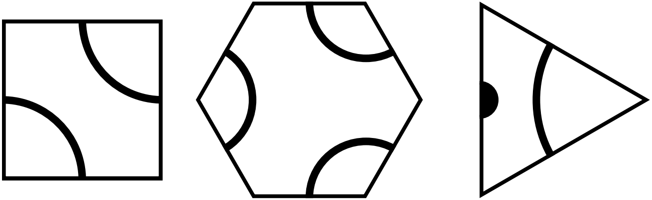

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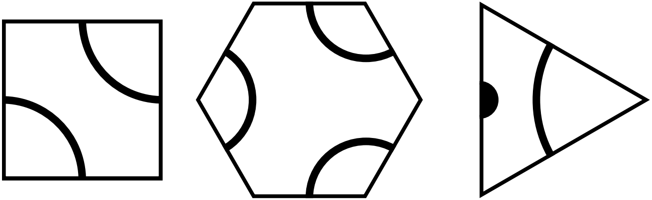

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- Just hinge a design with these tiles; add more such tiles when in the "open" position. Repeat.

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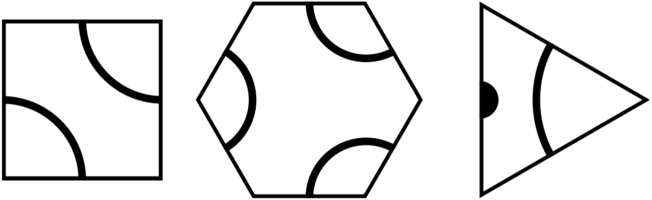

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- Example programs at: `www.mathamaze.co.uk/Truchet3/`

Binary operation sequences; continuous vs discrete

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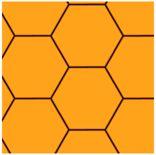
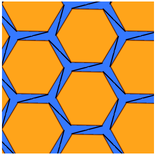
- The image at stage $t = 0, 1, 2, 3$ would be the appearance after **discrete replacements** corresponding to strings: $\varnothing$, 0, 00, 001, etc
- Image at a non integer value of t corresponds to an **intermediate** position; i.e., **continuous** interpolation of the discrete replacement rule by the hinging process.

Hexagon case

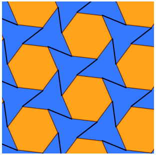
- Continuous process:



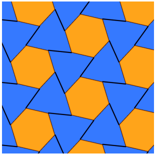
$t = 0$



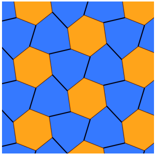
$t = 0.1$



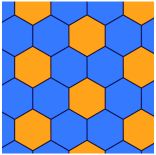
$t = 0.3$



$t = 0.5$



$t = 0.8$



$t = 1$

Hexagon case

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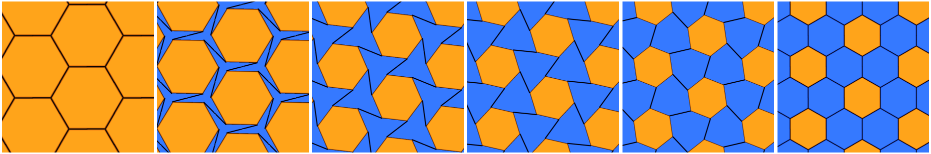

$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

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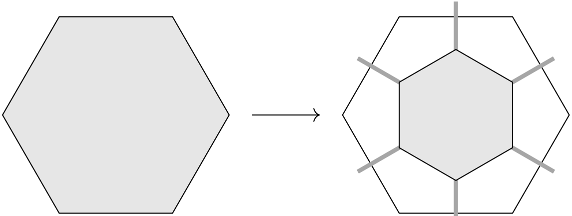

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Hexagon case

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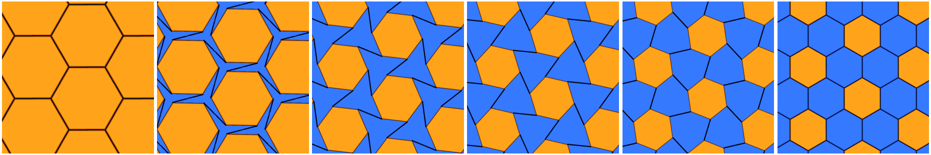

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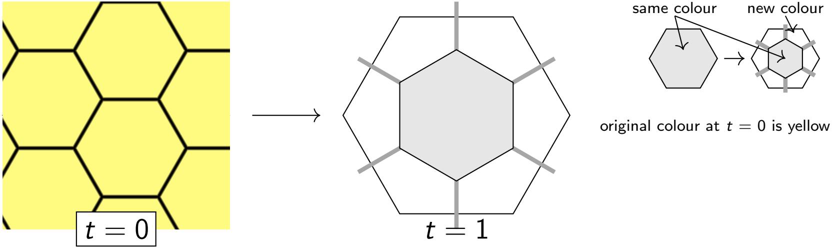

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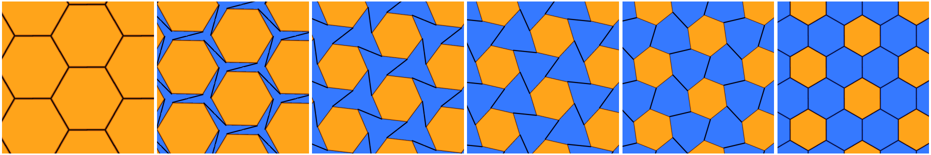

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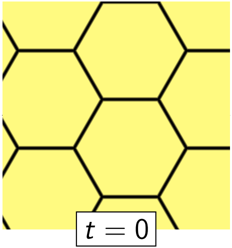
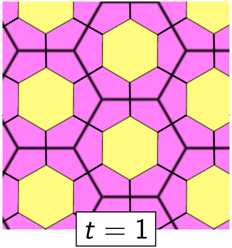
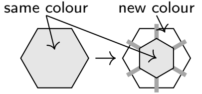


$\longrightarrow$

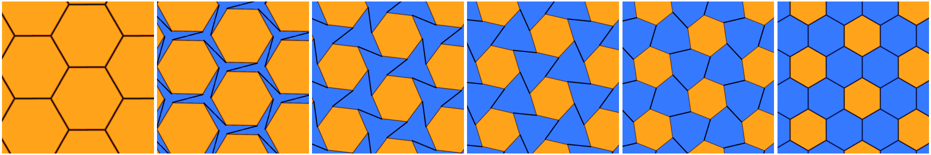




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink

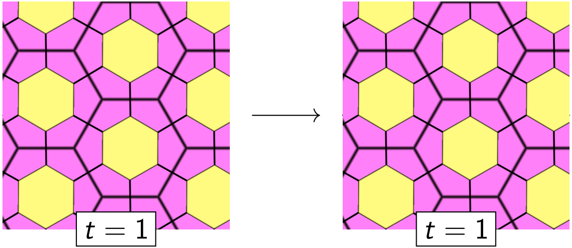
Hexagon case

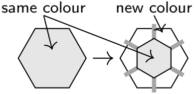
- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:

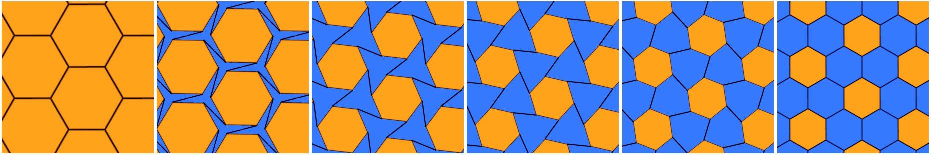




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink

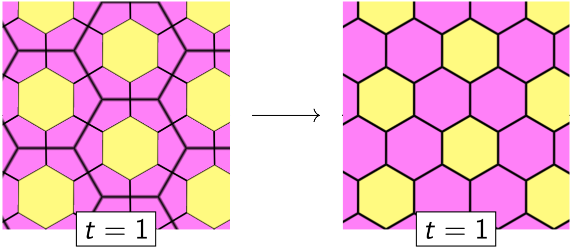
Hexagon case

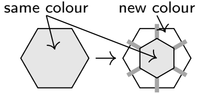
- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:

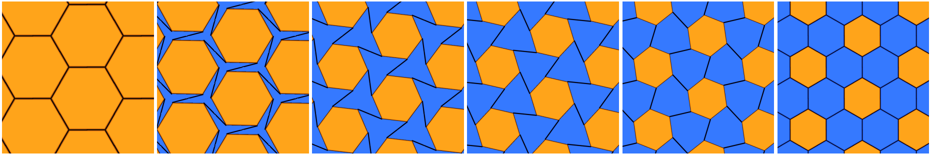




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink

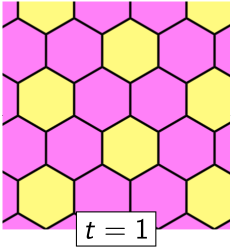
Hexagon case

- Continuous process:

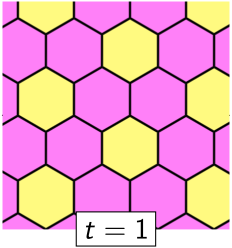


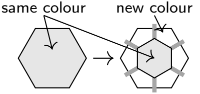
$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:



$\longrightarrow$

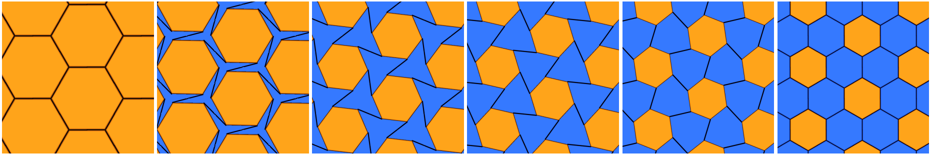




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink

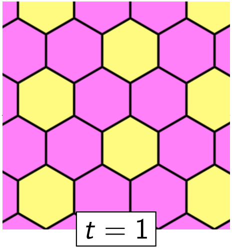
Hexagon case

- Continuous process:

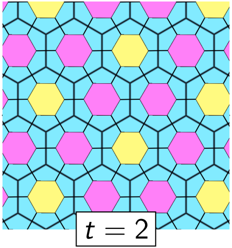


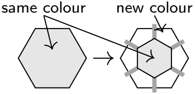
$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:



→

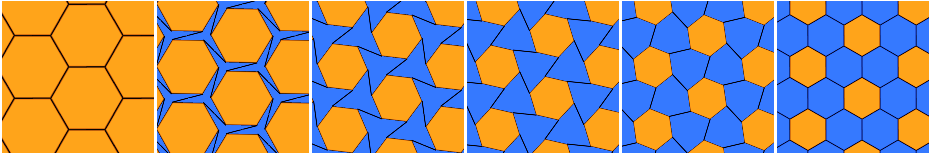




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink
new colour at $t = 2$ is blue

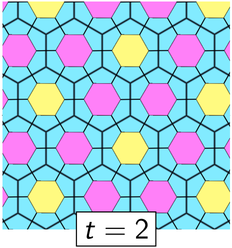
Hexagon case

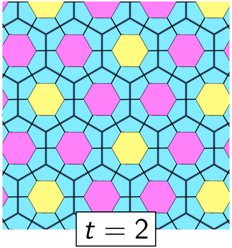
- Continuous process:

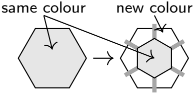


$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:



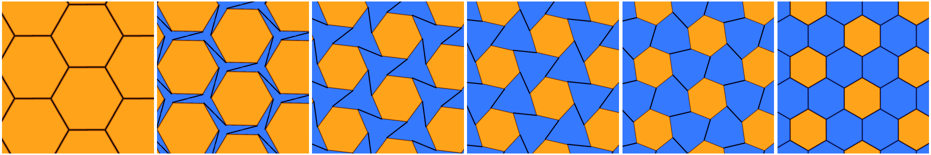




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink
new colour at $t = 2$ is blue

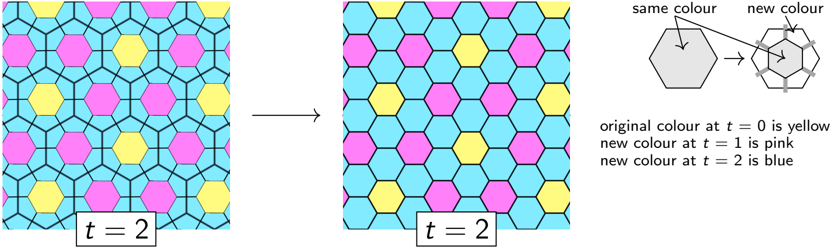
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

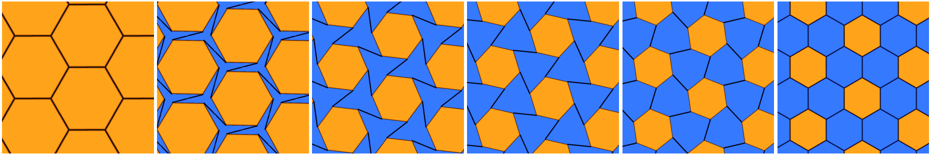
- Discrete process:



original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink
new colour at $t = 2$ is blue

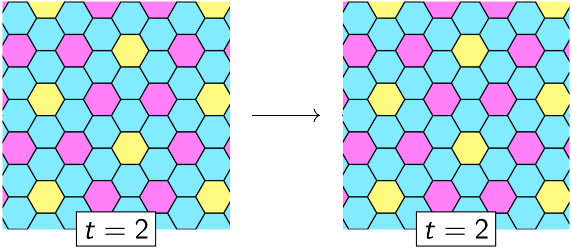
Hexagon case

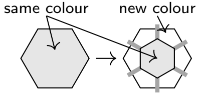
- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:

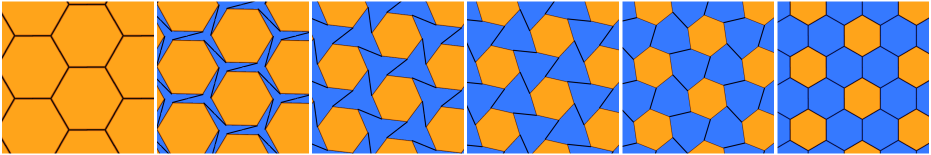




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink
new colour at $t = 2$ is blue

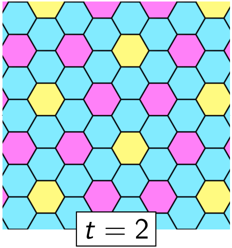
Hexagon case

- Continuous process:

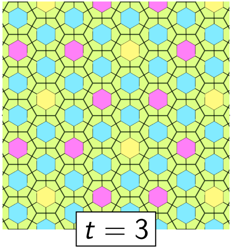


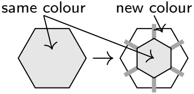
$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:



$\longrightarrow$

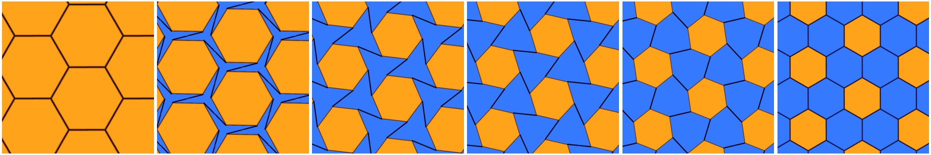




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink
new colour at $t = 2$ is blue
new colour at $t = 3$ is green

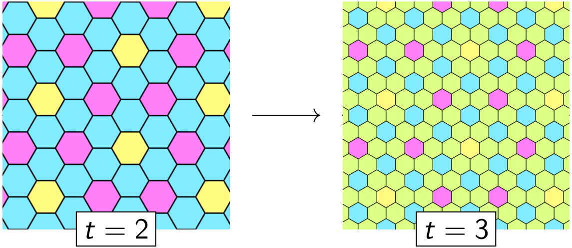
Hexagon case

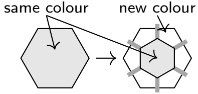
- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

- Discrete process:

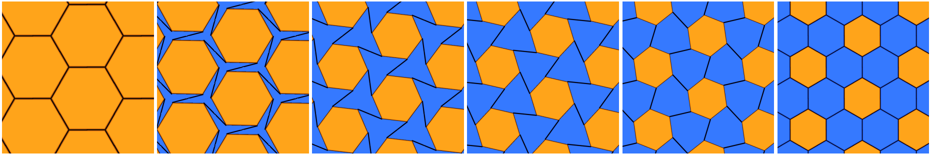




original colour at $t = 0$ is yellow
new colour at $t = 1$ is pink
new colour at $t = 2$ is blue
new colour at $t = 3$ is green

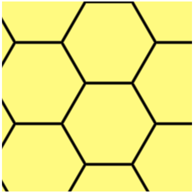
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

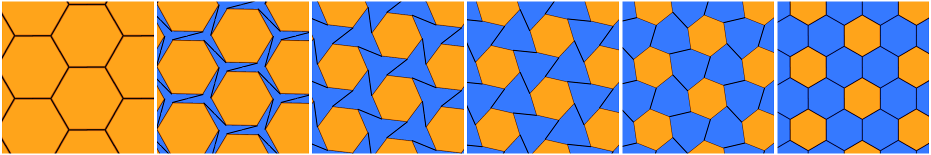
- Discrete process:



$t = 0$

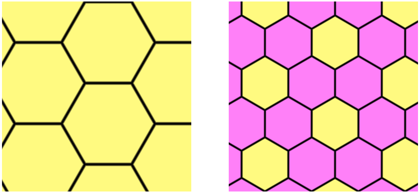
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

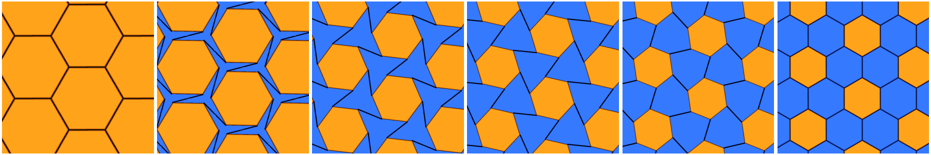
- Discrete process:



$t = 0$ $t = 1$

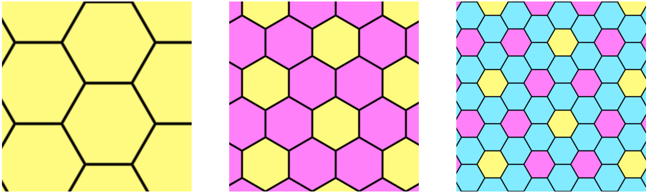
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

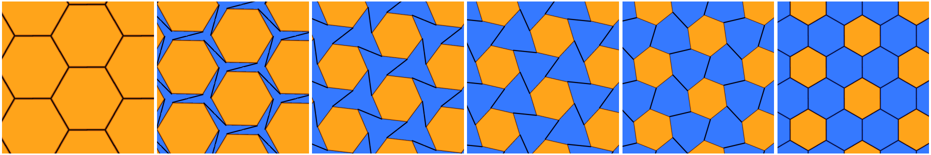
- Discrete process:



$t = 0$ $t = 1$ $t = 2$

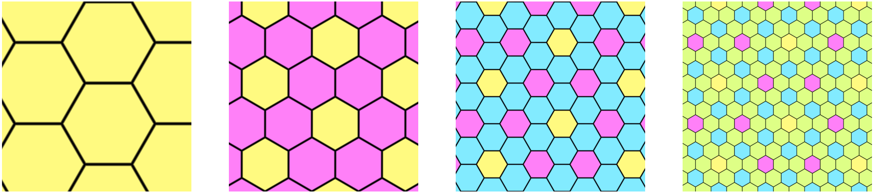
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

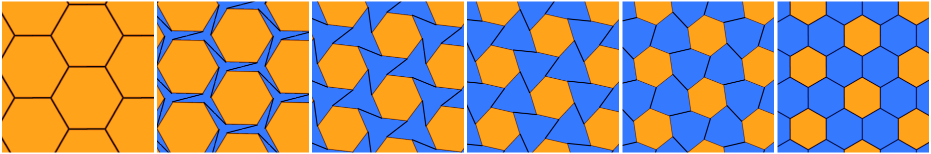
- Discrete process:



$t = 0$ $t = 1$ $t = 2$ $t = 3$

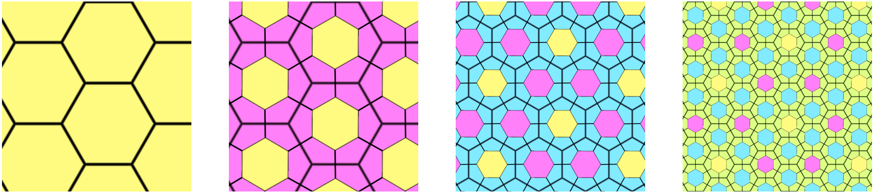
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

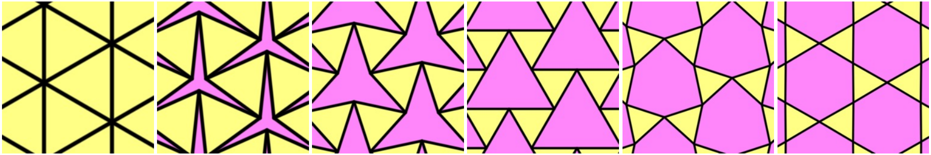
- Discrete process: (with lines to show replaced tiles)



$t = 0$ $t = 1$ $t = 2$ $t = 3$

Triangle 1 case

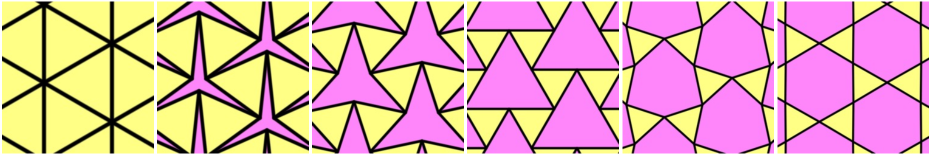
- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

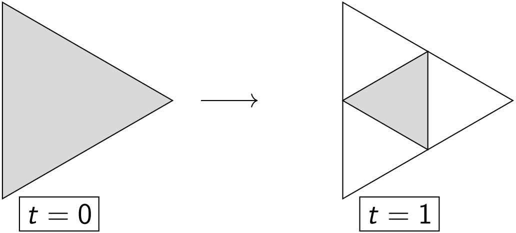
Triangle 1 case

- Continuous process:



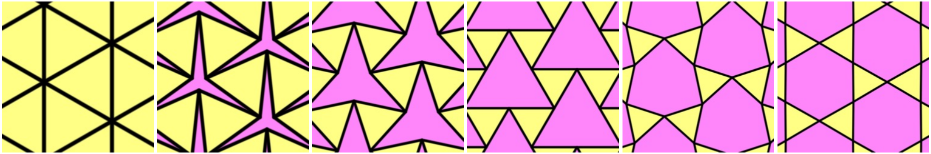
$t = 0$	$t = 0.1$	$t = 0.3$	$t = 0.5$	$t = 0.8$	$t = 1$

- Discrete process:



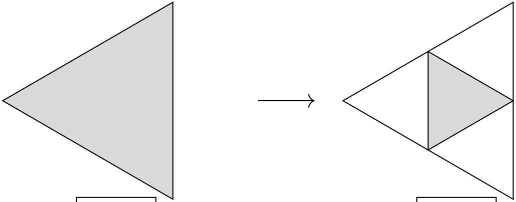
Triangle 1 case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

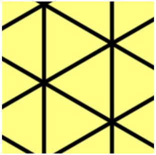
- Discrete process: (there are two orientations of triangles)

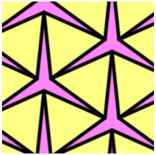


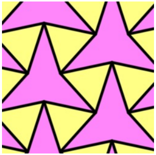
$t = 0$ $\longrightarrow$ $t = 1$

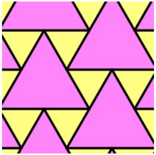
Triangle 1 case

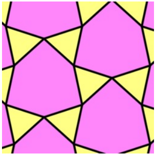
- Continuous process:

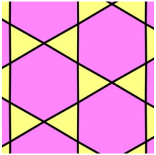

$t = 0$


$t = 0.1$

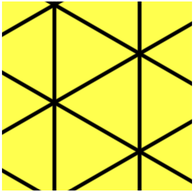

$t = 0.3$

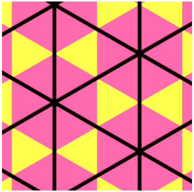

$t = 0.5$

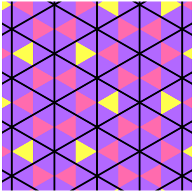

$t = 0.8$

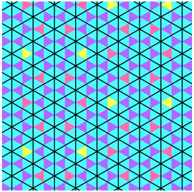

$t = 1$

- Discrete process:


$t = 0$

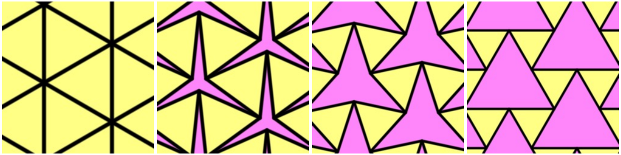

$t = 1$


$t = 2$


$t = 3$

Triangle 2 case

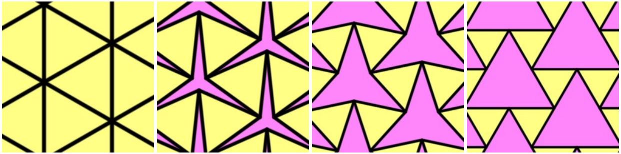
- Continuous process:



$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

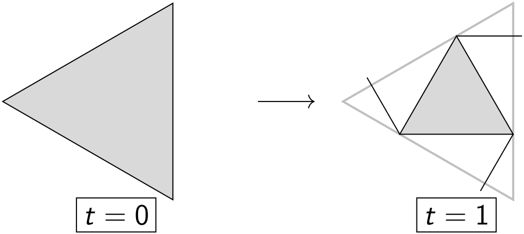
Triangle 2 case

- Continuous process:



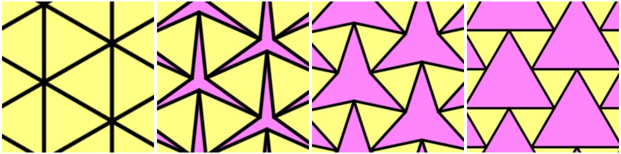
$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

- Discrete process: (also mirror image of this triangle)



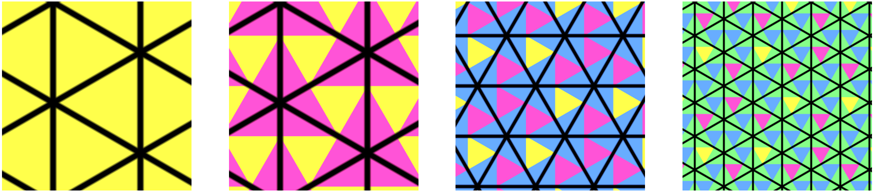
Triangle 2 case

- Continuous process:



$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

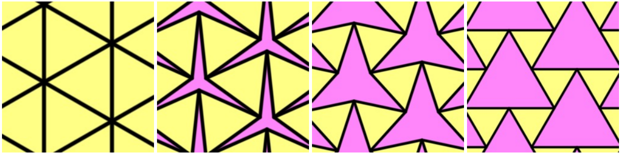
- Discrete process: (note original tile boundaries are not boundaries after operation)



$t = 0$ $t = 1$ $t = 2$ $t = 3$

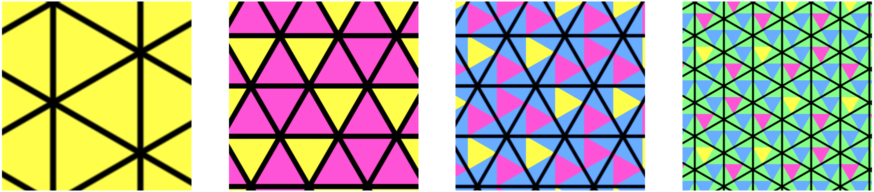
Triangle 2 case

- Continuous process:



$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

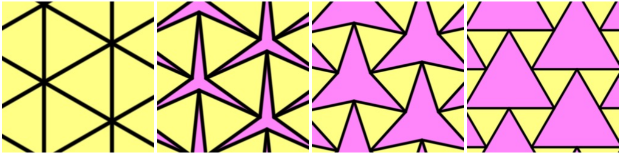
- Discrete process: (note original tile boundaries are not boundaries after operation)



$t = 0$ $t = 1$ $t = 2$ $t = 3$

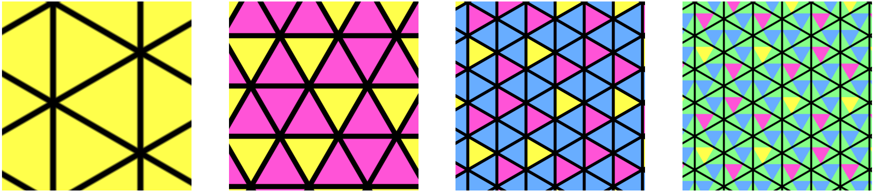
Triangle 2 case

- Continuous process:



$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

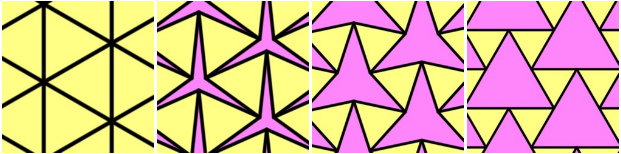
- Discrete process: (note original tile boundaries are not boundaries after operation)



$t = 0$ $t = 1$ $t = 2$ $t = 3$

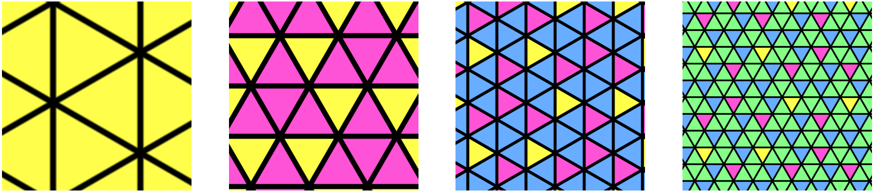
Triangle 2 case

- Continuous process:



$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

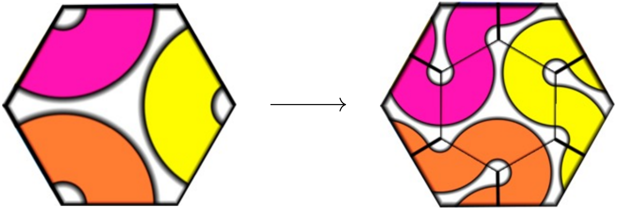
- Discrete process: (note original tile boundaries are not boundaries after operation)



$t = 0$ $t = 1$ $t = 2$ $t = 3$

Now add the Truchet / Smith designs

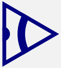
- Hexagons: (individual tile, one operation)



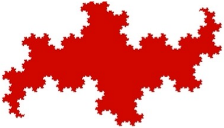
Now add the Truchet / Smith designs

- Hexagons: (several tiles, several operations)

$t = 0$ $t = 0.4$ $t = 1$ $t = 1.6$ $t = 2$

Now add the Truchet / Smith designs  

- Hexagons: (many operations ⇒ Fractiles! (terdragon))



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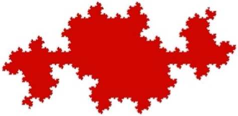

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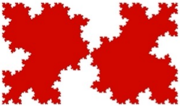

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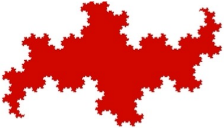
Now add the Truchet / Smith designs

- Hexagons:



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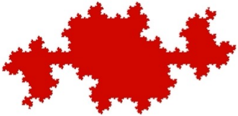

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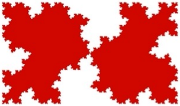

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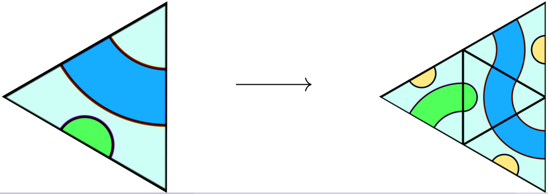
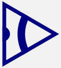
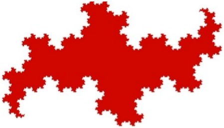
- Triangles, case 1: (individual tile, one operation)



Now add the Truchet / Smith designs

- Hexagons:



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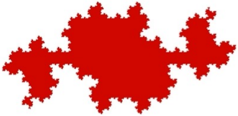

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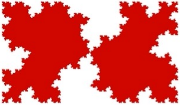

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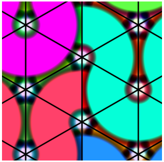
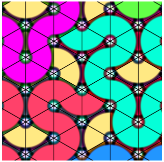
- Triangles, case 1: (several tiles, several operations)

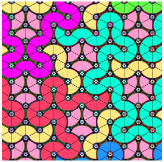
$t = 0$



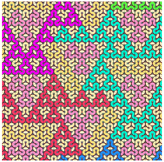
$t = 1$



$t = 2$



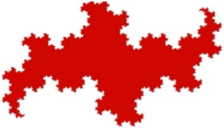
$t = 3$



$t = 4$

Now add the Truchet / Smith designs

- Hexagons:



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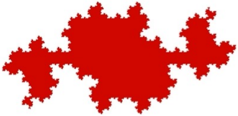

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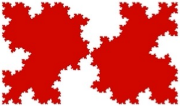

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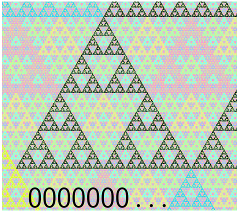
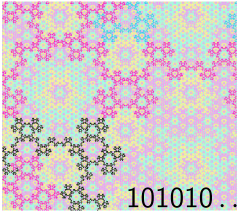
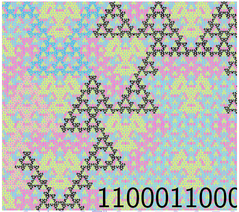
- Triangles, case 1: (many operations ⇒ Sierpinski variations)

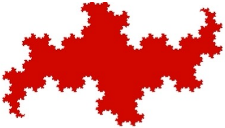






Now add the Truchet / Smith designs

- Hexagons:



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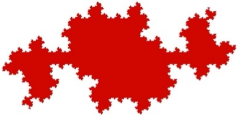

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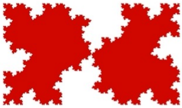

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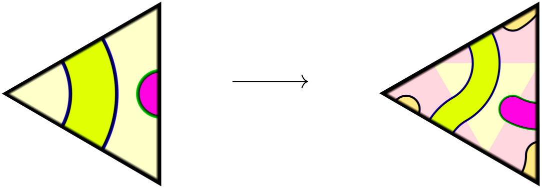
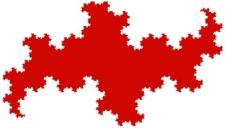
- Triangles, case 2: (individual tile, one operation)



Now add the Truchet / Smith designs

- Hexagons:



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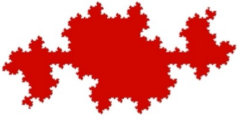

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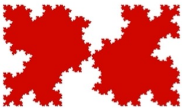

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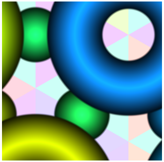
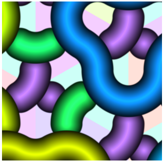
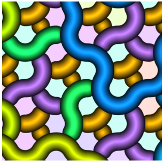
- Triangles, case 2: (several tiles, several operations)



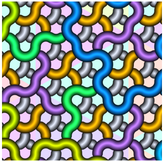
$t = 0$



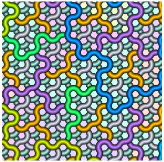
$t = 1$



$t = 2$

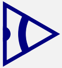


$t = 3$

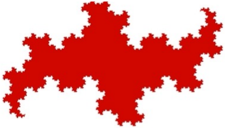


$t = 4$

Now add the Truchet / Smith designs



- Hexagons:



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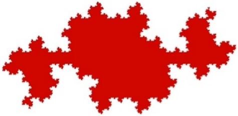

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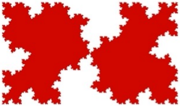

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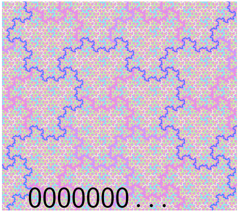
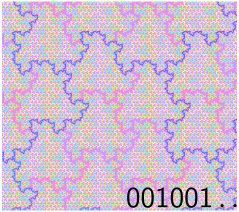
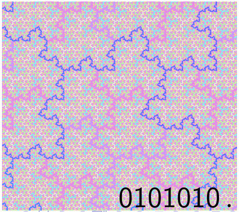
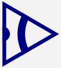
- Triangles, case 2: (many operations ⇒ terdragon boundary)



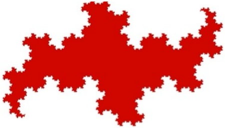




Now add the Truchet / Smith designs

- Hexagons:



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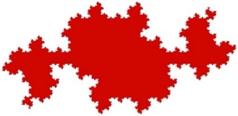

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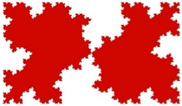

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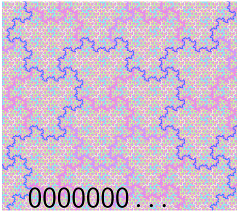
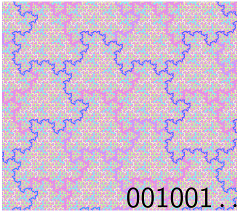
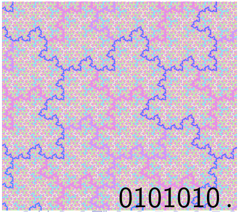
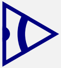
- Triangles, case 2: (many operations ⇒ fudge flake)

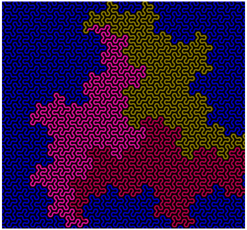
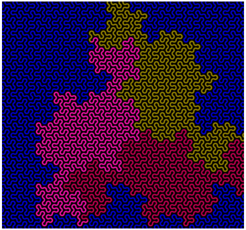
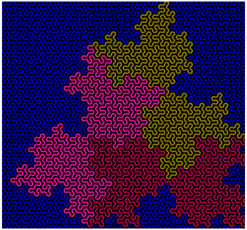




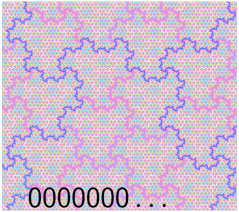


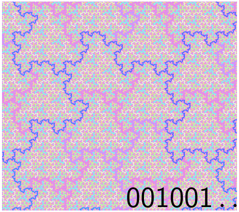
Now add the Truchet / Smith designs  

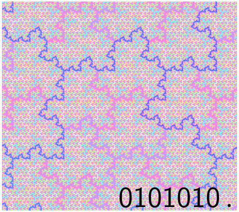
- Hexagons:

- Triangles, case 2: (many operations ⇒ fudge flake)

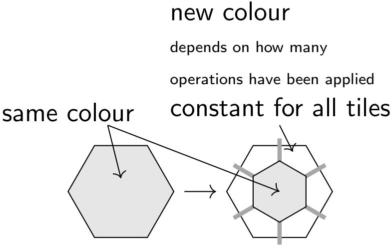




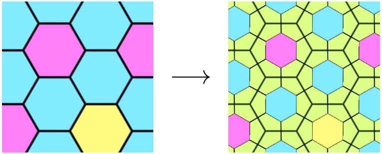


Digression: Fun with undecorated hinged hexagon tiles

- How should we colour the tiles? <u>“correct”</u>:

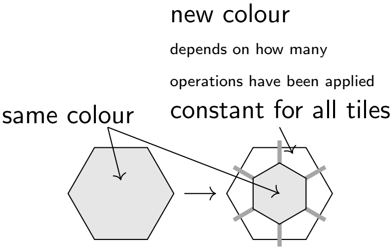


example: new colour is green

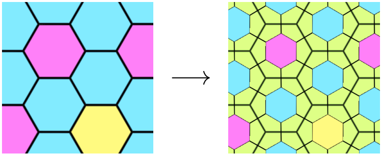


Digression: Fun with undecorated hinged hexagon tiles

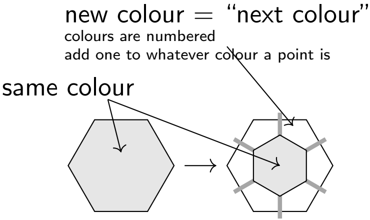
- How should we colour the tiles? "correct":



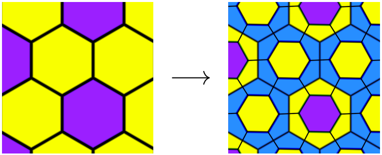
example: new colour is green



- "wrong":



(0) purple; (1) yellow; (2) blue



Digression: Fun with undecorated hinged hexagon tiles

- How should we colour the tiles? "correct":

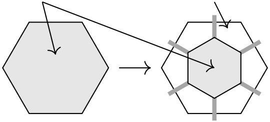
new colour

depends on how many

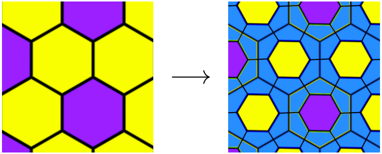
operations have been applied

constant for all tiles

same colour



example: new colour is blue



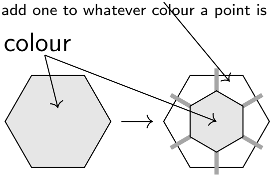
- "wrong":

new colour = "next colour"

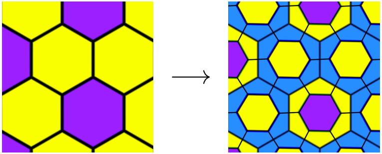
colours are numbered

add one to whatever colour a point is

same colour



(0) purple; (1) yellow; (2) blue



Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:

Koch snowflake

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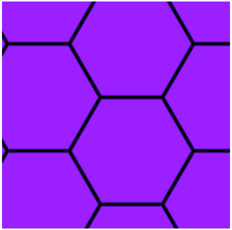
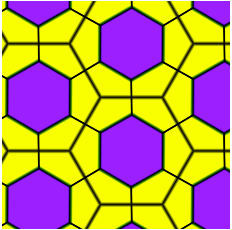
-



$t = 0$

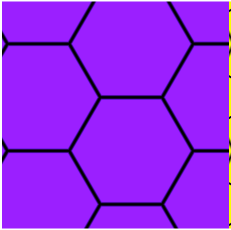
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
- 

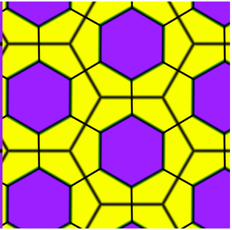
$t = 1$

Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
-

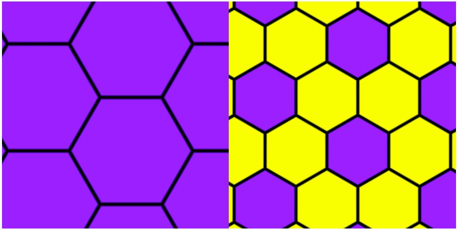


$t = 0$



$t = 1$

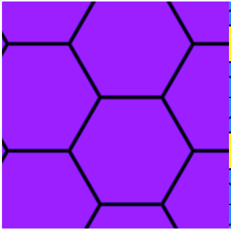
Koch snowflake

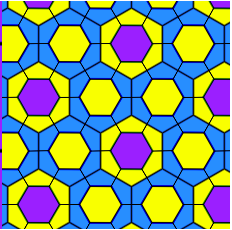
- Let's apply the "wrong" "colour + 1" rule:
- 

$t = 0$ $t = 1$

Koch snowflake

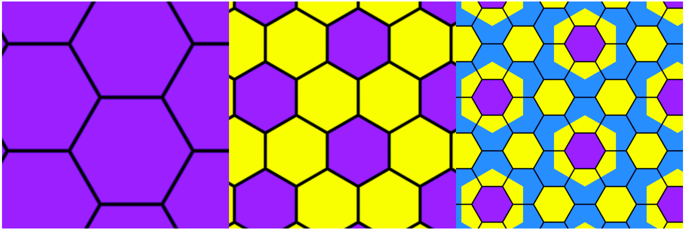
- Let's apply the "wrong" "colour + 1" rule:
-





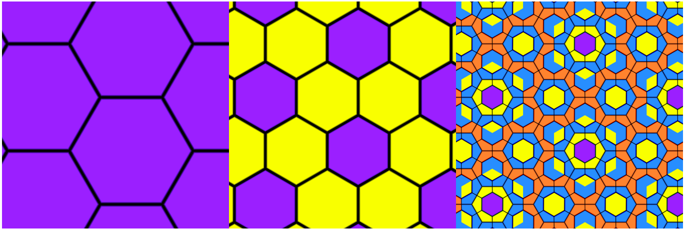
$t = 0$ $t = 2$

Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
- 

$t = 0$ $t = 1$ $t = 2$

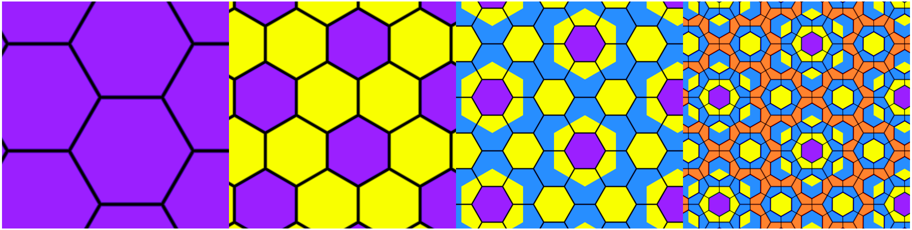
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
- 

$t = 0$ $t = 1$ $t = 3$

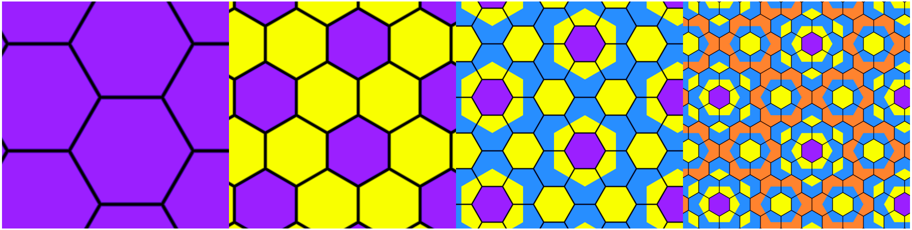
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
-



$t = 0$ $t = 1$ $t = 2$ $t = 3$

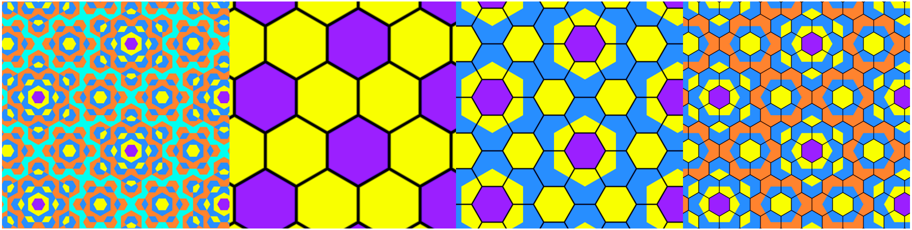
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
- 

$t = 0$ $t = 1$ $t = 2$ $t = 3$

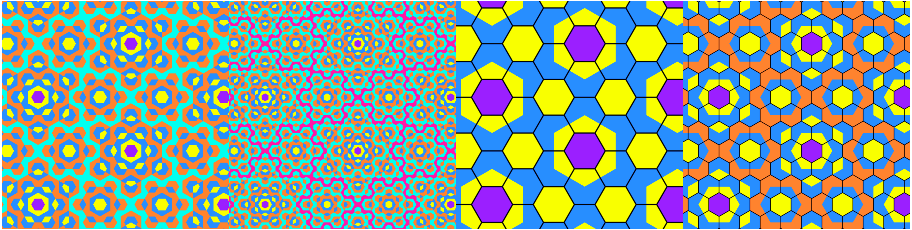
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
-



$t = 4$ $t = 1$ $t = 2$ $t = 3$

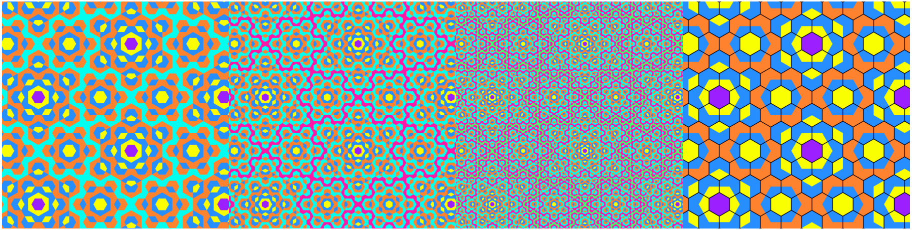
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
- 

$t = 4$ $t = 5$ $t = 2$ $t = 3$

Koch snowflake

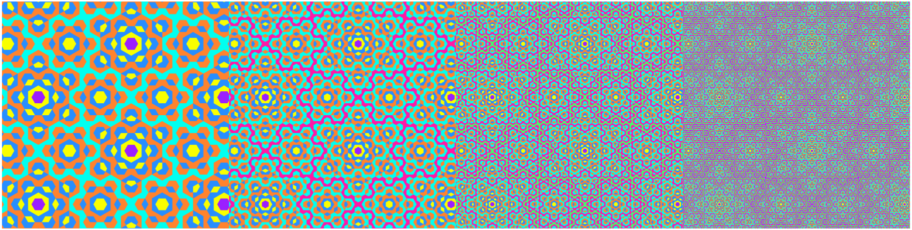
- Let's apply the "wrong" "colour + 1" rule:
-



$t = 4$ $t = 5$ $t = 6$ $t = 3$

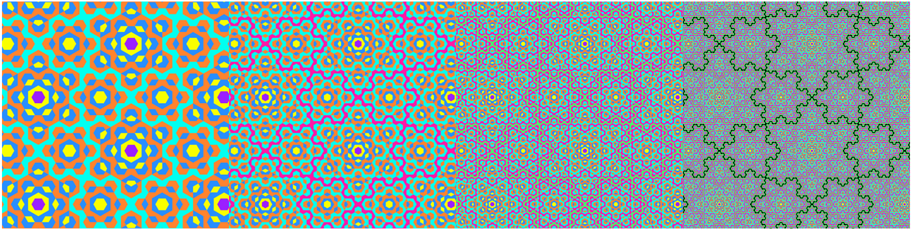
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
-



$t = 4$ $t = 5$ $t = 6$ $t = 7$

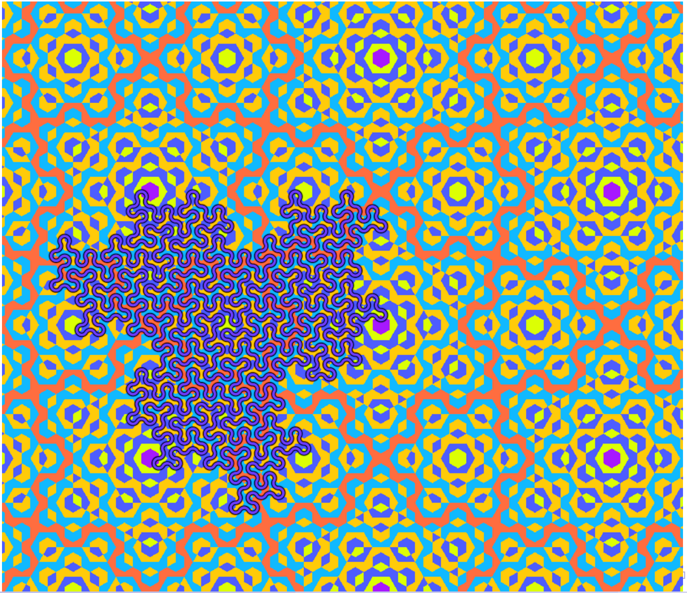
Koch snowflake

- Let's apply the "wrong" "colour + 1" rule:
- 

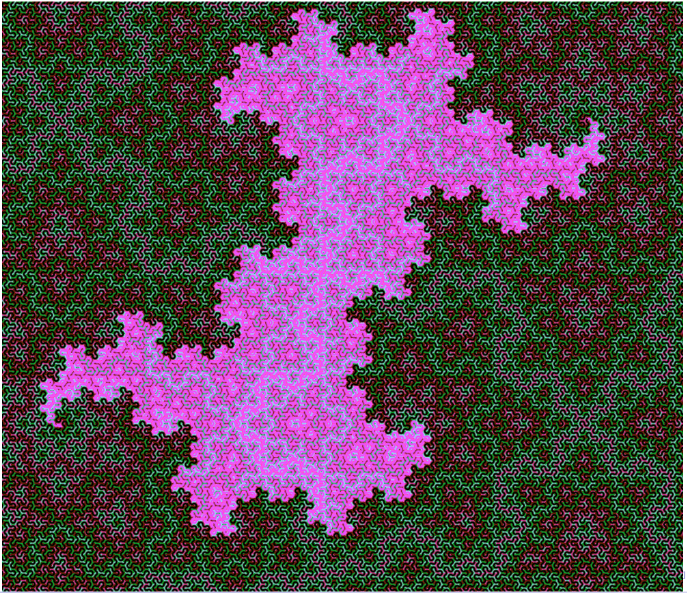
$t = 4$ $t = 5$ $t = 6$ $t = 7$

- A tessellation of Koch snowflakes appears

Terdragon and Koch snowflake examples



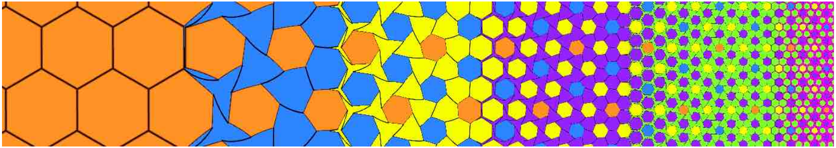
Terdragon and Koch snowflake examples



Left out; hints of ideas

No time to cover:

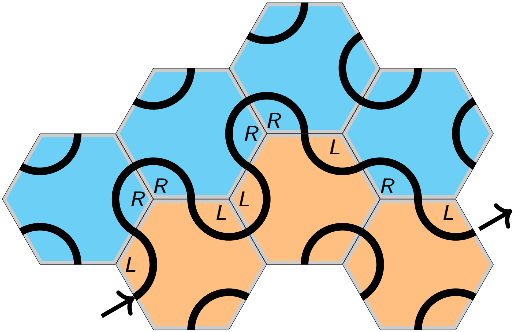
- Varying operation iteration level accross image



Left out; hints of ideas

No time to cover:

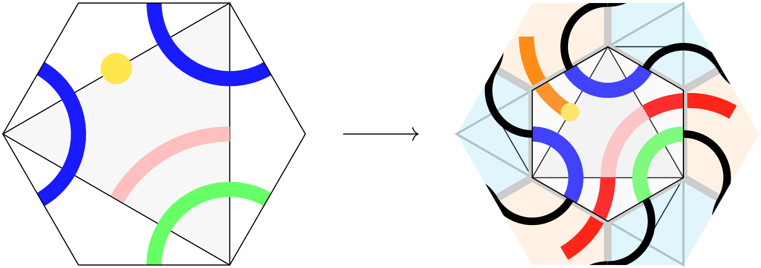
- Varying operation iteration level accross image
- L-systems, used to describe these fractals (replacement rule; symbols describe paths)



Left out; hints of ideas

No time to cover:

- Varying operation iteration level accross image
- L-systems, used to describe these fractals (replacement rule; symbols describe paths)
- How the terdragon and its boundary are related via the tilings



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No time to cover:

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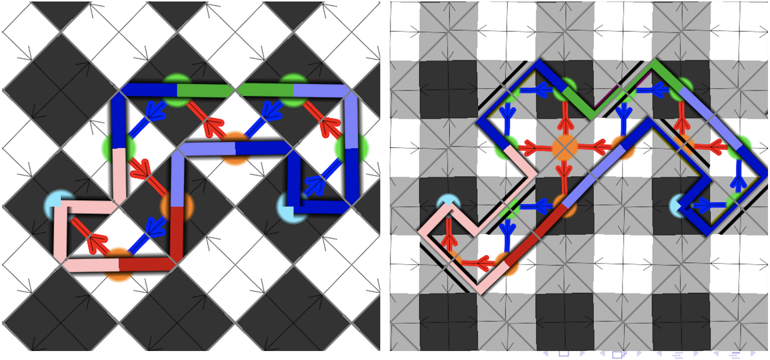
- How the terdragon and its boundary are related via the tilings
- Extention to get boundary L-systems for Heighway dragon and certain square grid based space filling curves



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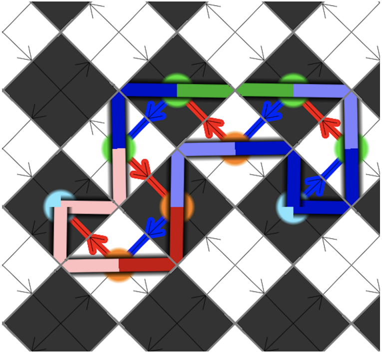

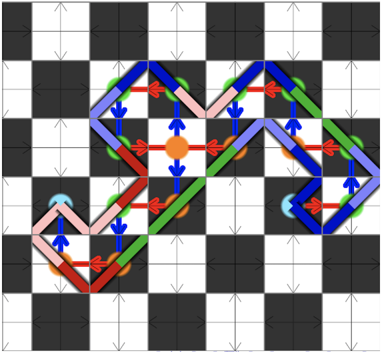

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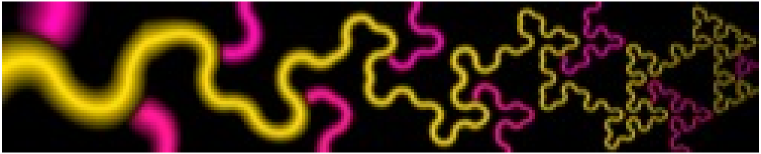
- You can read more at `https://www.mathamaze.co.uk/Truchet3/`



End of talk

Thank you for listening!