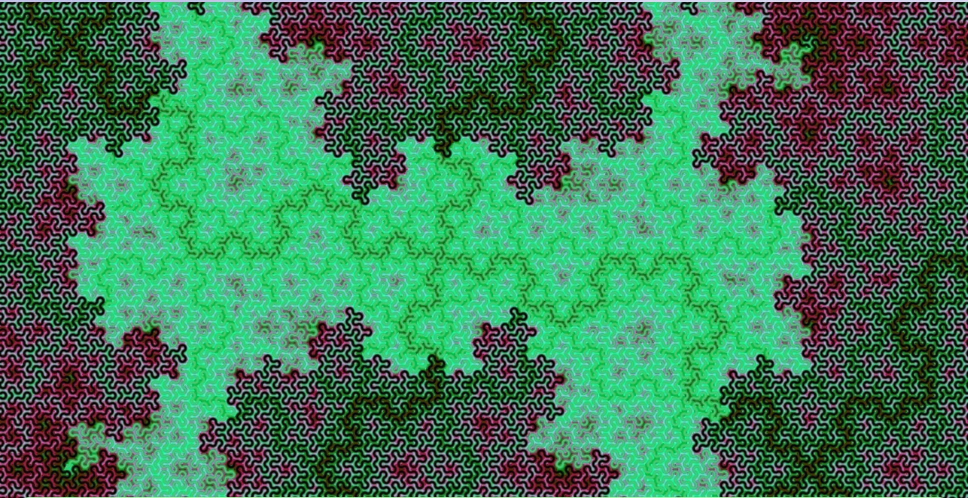
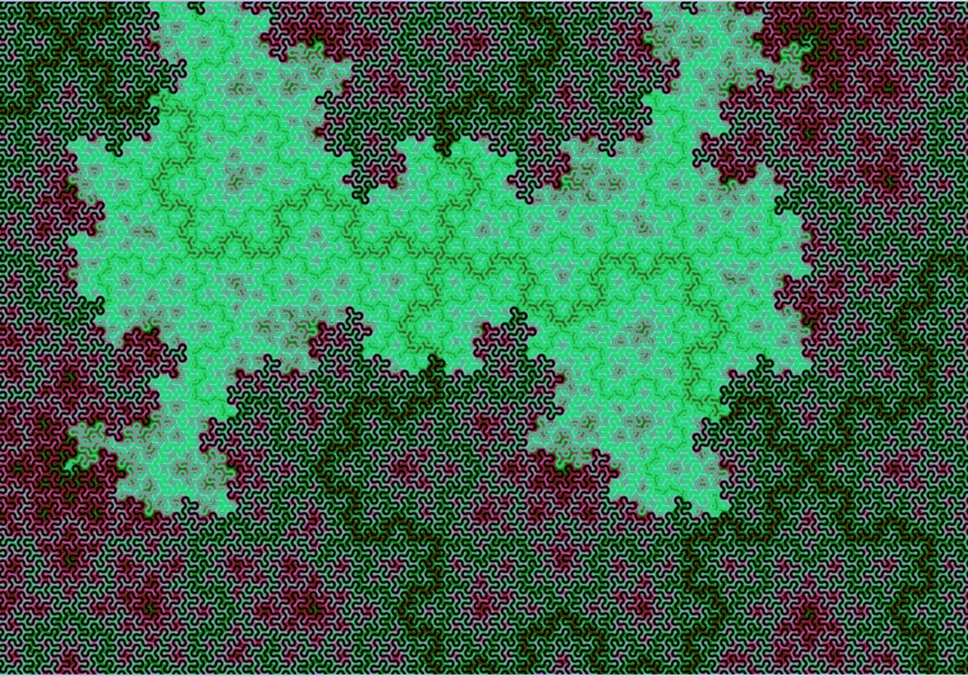


Fractals from Hinged Hexagon and Triangle Tilings

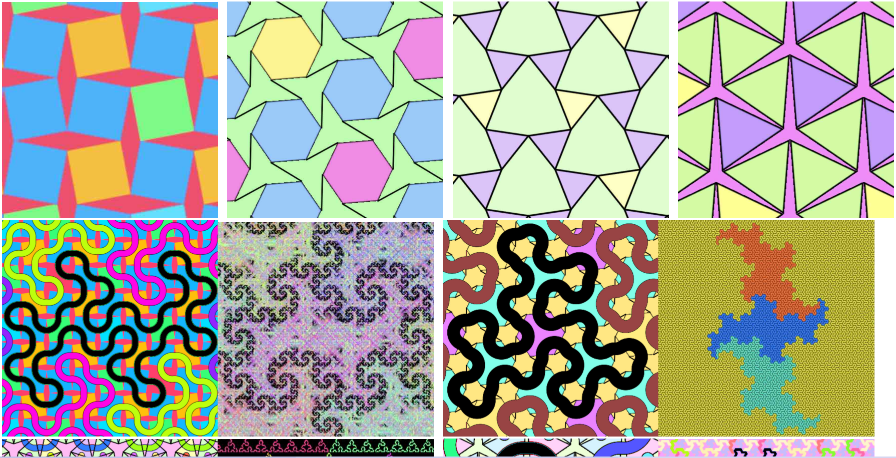
Helena Verrill, Warwick University, UK



Summary of Talk

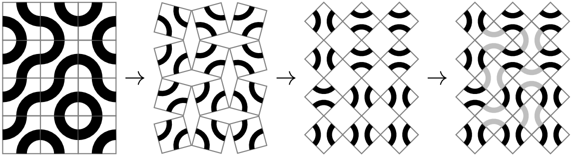
(Graphics from `https://www.mathamaze.co.uk/Truchet2/`)

Take an initial tiling, e.g., of squares, hexagons or triangles. Decorate each tile with arcs; hinge to obtain new tiling; repeat

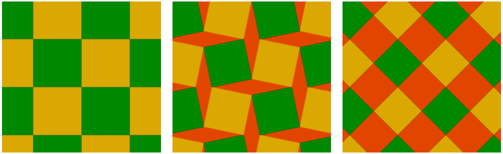


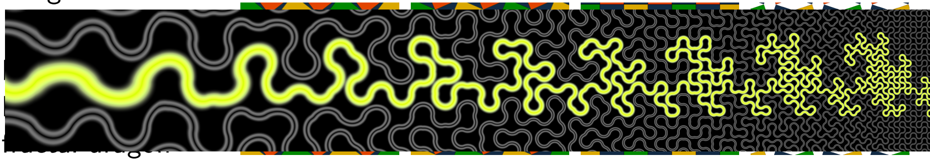
Recall from last year: Hinged squares

Hinged
Truchet tiling

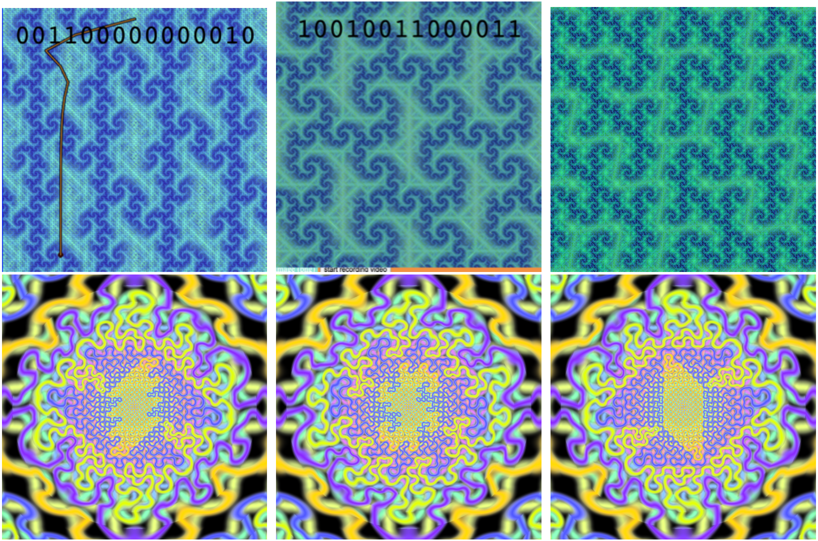


Hinged tiling;
rotate 45°;
scale by $\sqrt{2}$;
background
becomes
foreground



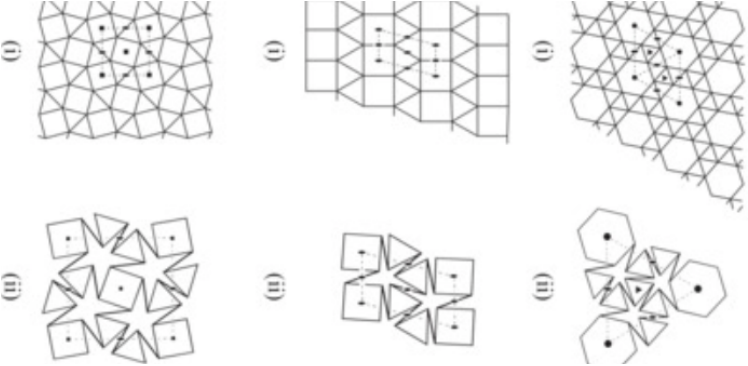


(this is from last year's talk; spot Heighway's dragon)



What about other hinged tilings?

- There are many hinged tilings
- E.g., if you want to try this at home, see e.g. T. Tarnai, P. Fowler, S. Guest and F. Kovacs. "Equiauxetic Hinged Archimedean Tilings." Symmetry (Basel),vol, 14 (2), 2022. https://www.mdpi.com/2073-8994/14/2/232

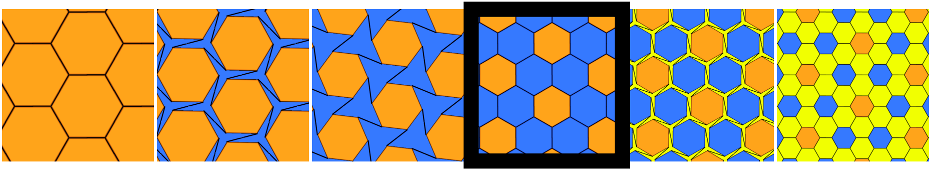


or

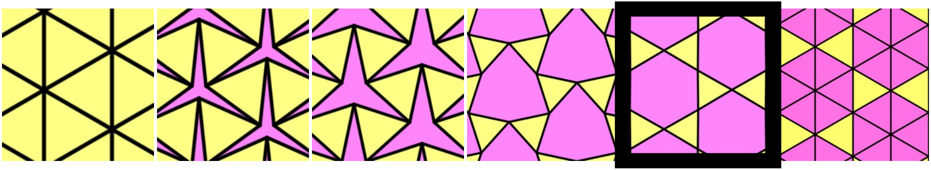
Alfinio Flores, "Hinged tilings" page, https://www.public.asu.edu/~aaaafp/tiling/hingedtilingtext.html

The hinged tilings

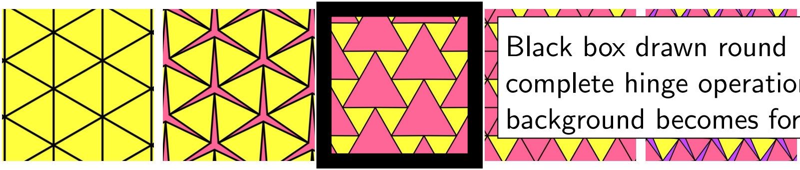
Hexagons with links: (rotate hexagons through 30°)



Triangles (1) (rotate triangles through 60°)



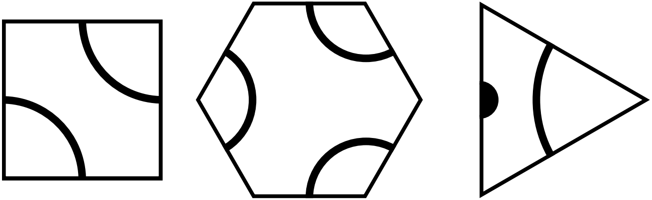
Triangles (2) (rotate triangles through 30°)



Black box drawn round complete hinge operation before background becomes foreground

Truchet designs and fractals

- We use the following tile designs:



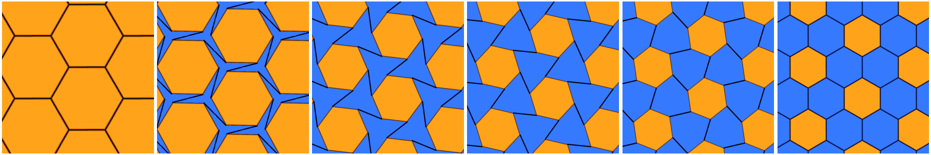
- Inspired by the Smith (Truchet) tile design (square case);
- Truchet had the idea of putting together a lot of identical tiles at different orientations; Smith had the idea of using circle arcs
- Just hinge a design with these tiles; add more such tiles when in the "open" position. Repeat.
- Example program at:

Binary operation sequences; continuous vs discrete

- Overview of next few slides:
- We have a **continuous hinging operation**, starting from closed, at $t = 0$, ending at "open", $t = 1$.
- ("open" is when you decided you've finished your hinging and want to replace background with new forground tiles etc; this is your choice; depending on what works)
- In each case, we can rotate **clockwise** or **counterclockwise**; denote this by 0 or 1. So, an operation sequence can be described as a **binary string** e.g. 001010111 etc.
- The image at stage $t = 0, 1, 2, 3$ would be the appearance after **discrete replacements** corresponding to strings: $\emptyset$, 0, 00, 001, etc
- Image at a non integer value of t corresponds to an **intermediate** position; i.e., continuous interpolation of the discrete replacement rule by the hinging process.

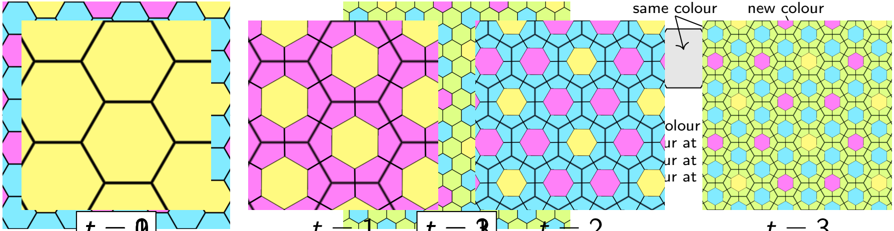
Hexagon case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

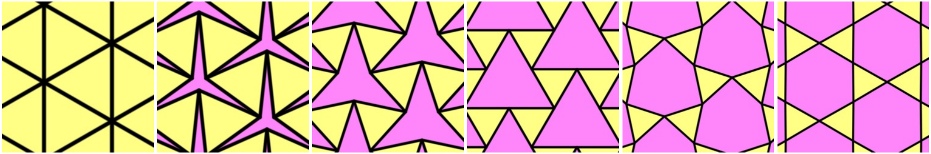
- Discrete process: (with lines to show replaced tiles)



$\boxed{t = 0}$ $t = 1$ $\boxed{t = 3}$ $t = 2$ $t = 3$

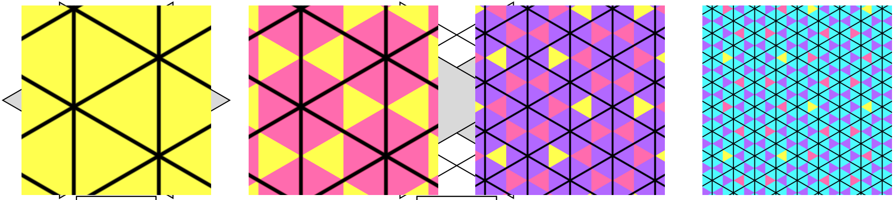
Triangle 1 case

- Continuous process:



$t = 0$ $t = 0.1$ $t = 0.3$ $t = 0.5$ $t = 0.8$ $t = 1$

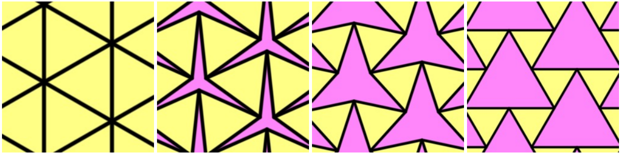
- Discrete process: (there are two orientations of triangles)



$\boxed{t = 0}$ $t = 1$ $\boxed{t = 1}$ $t = 2$ $t = 3$

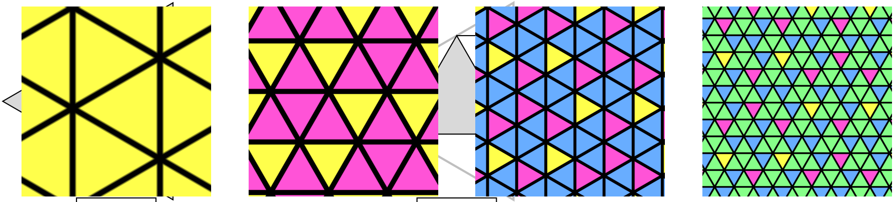
Triangle 2 case

- Continuous process:



$t = 0$ $t = 0.2$ $t = 0.6$ $t = 1$

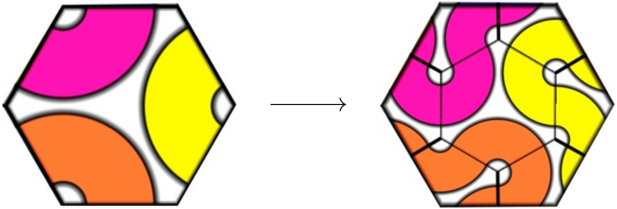
- Discrete process: (also mirror image of this triangle) (note original tile boundaries are not boundaries after operation)



$\boxed{t = 0}$ $t = 1$ $\boxed{t = 1}$ $t = 2$ $t = 3$

Now add the Truchet / Smith designs

- Hexagons: (individual tile, one operation) (several tiles, several operations) (many operations $\Rightarrow$ Fractiles! (terdragon))



$t = 0$ $t = 0.4$ $t = 1$ $t = 1.6$ $t = 2$

Digression: Fun with undecorated hinged hexagon tiles

- How should we colour the tiles? "correct":

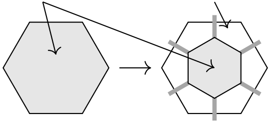
new colour

depends on how many

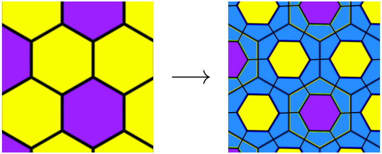
operations have been applied

same colour

constant for all tiles



example: new colour is green



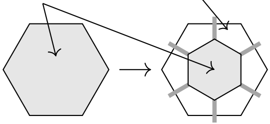
- "wrong":

new colour = "next colour"

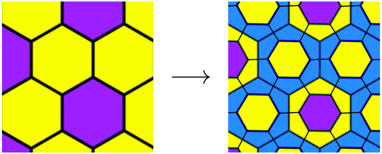
colours are numbered

add one to whatever colour a point is

same colour

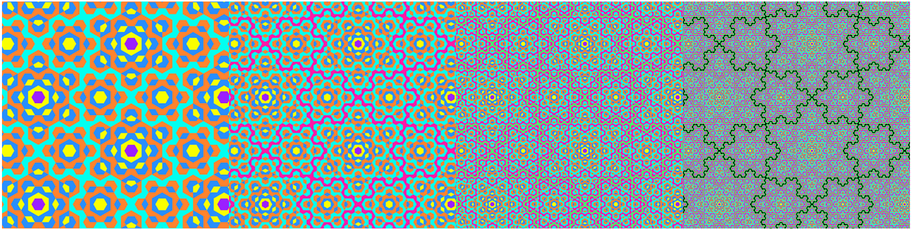


(0) purple; (1) yellow; (2) blue



Koch snowflake

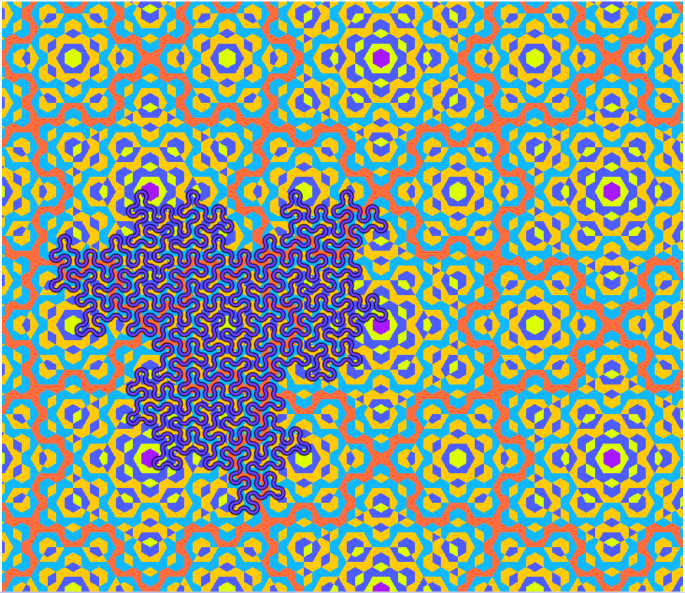
- Let's apply the "wrong" "colour + 1" rule:
-



$t = 4$ $t = 5$ $t = 6$ $t = 7$

- A tessellation of Koch snowflakes appears

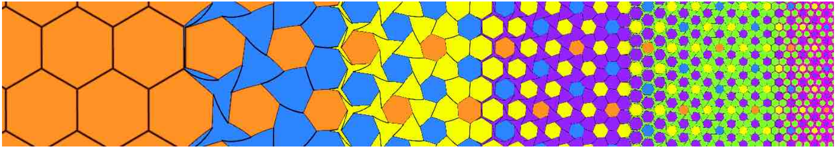
Terdragon and Koch snowflake examples



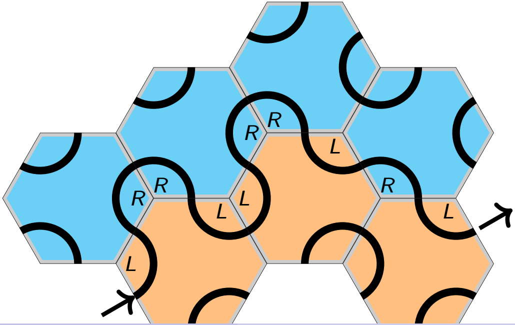
Left out; hints of ideas

No time to cover:

- Varying operation iteration level accross image



- L-systems, used to describe these fractals (replacement rule; symbols describe paths)



End of talk

Thank you for listening!