Dobble

The game Dobble was invented based on a finite projective plane. Junior Dobble is isomorphic to $\mathbb{P}^2(\mathbb{F}_5)$, and corresponds to a finite projective plane of order 6. Each line contains 6 points, each point is on 6 lines. In the game as sold, one card is missing.

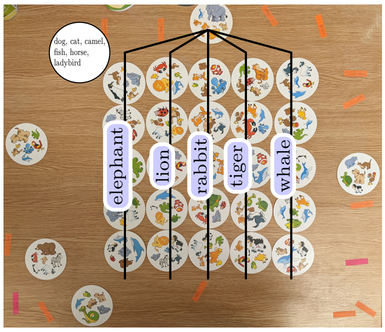
The missing card is:
{dog, cat, camel, fish, ladybird, horse}.

Each line corresponds to a different animal.

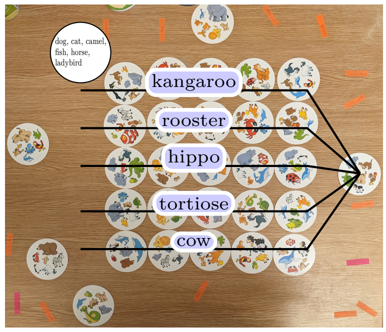
There are 31 different animal symbols, one for each line in the projective space $\mathbb{P}^2(\mathbb{F}_5)$.
I have made the following choice of animal to point of $\mathbb{P}^2(\mathbb{F}_5)$. A different choice would correspond to an element of $PGL(3, \mathbb{F}_5)$, which has order 372000.

animal	orthogonal vector	animal	orthogonal vector
dog	(0:0:1)	duck	(1:1:1)
elephant	(1:0:0)	parrot	(1:1:2)
lion	(4:0:1)	crocodile	(1:1:3)
rabbit	(2:0:1)	octopus	(1:1:4)
tiger	(3:0:1)	shark	(1:1:0)
whale	(1:0:1)	penguin	(3:1:1)
kangaroo	(0:1:1)	zebra	(3:1:2)
rooster	(0:3:1)	crab	(3:1:3)
hippo	(0:2:1)	sheep	(3:1:4)
tortoise	(0:4:1)	bear	(3:1:0)
cow	(0:0:1)	horse	(2:1:1)
dolphin	(4:1:0)	fish	(2:1:2)
gorilla	(4:1:1)	cat	(2:1:3)
snake	(4:1:2)	camel	(2:1:4)
frog	(4:1:3)	ladybird	(2:1:0)
owl	(4:1:1)		

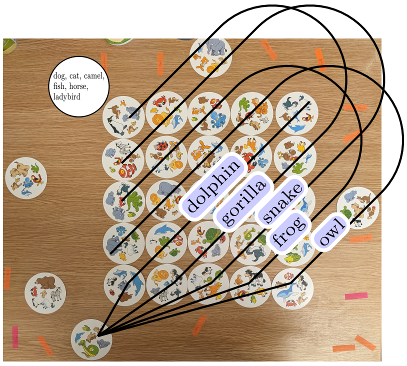
E.g. the "cat" line, consists of points $(x : y : z)$ with $y = 2z - 2x$, i.e., $(x, y, z) \cdot (2, 1, -2) = 0$ meaning "cat" is the line orthogonal to $(2 : 1 : -2) \equiv (2 : 1 : 3) \equiv (4 : 2 : 1)$.
You can work out the point on $\mathbb{P}^2(\mathbb{F}_5)$ corresponding to your card by just considering two of the animals on it. E.g., the card containing gorilla $(4 : 1 : 1)$ and cat $(2 : 1 : 3)$ must be a point $(x : y : z)$ with $4x + y + z \equiv 2x + y + 3z \equiv 0 \mod 5$. From this we get $x \equiv y + z$ (from first condition) and then $2y + 2z + y + 3z \equiv 3y \equiv 0$ (sub in second) so $y \equiv 0$, and $x \equiv z$, so (up to scaling) the point is $(1 : 0 : 1)$.



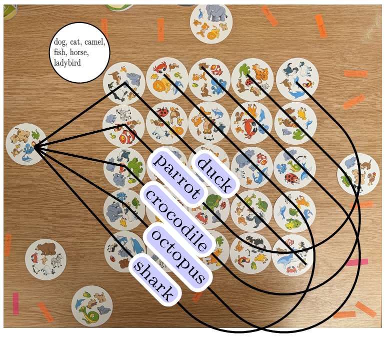
lines of the form $\{(\alpha s : t : s) : (t : s) \in \mathbb{P}^1\}$



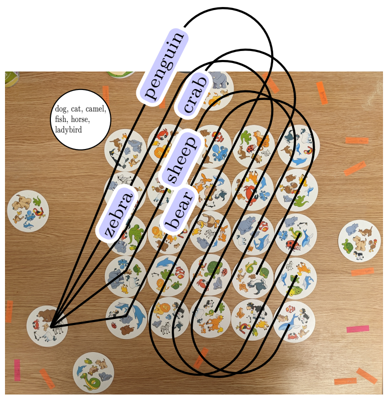
lines of the form $\{(t : \alpha s : s) : (t : s) \in \mathbb{P}^1\}$



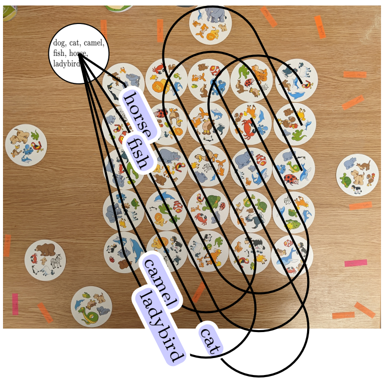
lines of the form $\{(t : t + \alpha s : s) : (t : s) \in \mathbb{P}^1\}$



lines of the form $\{(t : \alpha s - t : s) : (t : s) \in \mathbb{P}^1\}$



lines of the form $\{(t : 2t + \alpha s : s) : (t : s) \in \mathbb{P}^1\}$



lines of the form $\{(t : \alpha s - 2t : s) : (t : s) \in \mathbb{P}^1\}$

$(0 : 1 : 0)$

$(1 : -2 : 0)$ (0,4) (1,4) (2,4) (3,4) (4,4)

(0,3) (1,3) (2,3) (3,3) (4,3)

$(1 : -1 : 0)$ (0,2) (1,2) (2,2) (3,2) (4,2) $(1 : 0 : 0)$

(0,1) (1,1) (2,1) (3,1) (4,1)

$(1 : 2 : 0)$ (0,0) (1,0) (2,0) (3,0) (4,0)

$(1 : 1 : 0)$

interpret (x, y) as $(x : y : 1)$; all computations mod 5

This shows the 6 families of "parallel" lines. Note that "parallel" here means not meeting in the copy of $\mathbb{F}_5^2 \subset \mathbb{P}^2(\mathbb{F}_5)$ given by $(x, y) \mapsto (x : y : 1)$.