FLAT HERRING BONE ORIGAMI TESSELLATION

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0.1. **starting point.** This is my first attempt of how to fold a herring bone tessellation. I thought it might be interesting to describe how I came up with this, since I'll certainly forget myself if I don't write it down!

At the BOS origami convention on 7 April 2019, I showed some origami tessellations, including the one in Figure 1. This fold comes from taking a tessellation, as in [Verrill, figure 14], (many more references in e.g., [Frederickson, page 117] the crease pattern is in the middle; this is a hinged version of the tessellation on the right of the figure. Because of Kawasaki-Justin theorem (see e.g., [Hull, p173], since the alternate angles at a vertex add up to 180° (by construction, due to the sums of angles in a parallellogram), there is a chance it will fold flat. And it turns out it does. Incidentally, it also has the same appearance on both sides.

0.2. **challenge.** Assia Brill, who is also a very accomplished knitter, asked if I could make it into a better "herring bone" pattern, that is, the lines of the pattern, marked in bold red, on the left diagram Figure 2 should not be there, it should look like the edited picture in the middle.

I thought that was an interesting question, so I got to work on finding a better herring bone origami tessellation. This is what I have so far.

0.3. **Flatness.** Note, a quick search on google comes up with lots of beautiful pictures of Herring bone tessellations, but these all seem to be three dimensional. What I was looking for here was a two dimensional, flat origami.

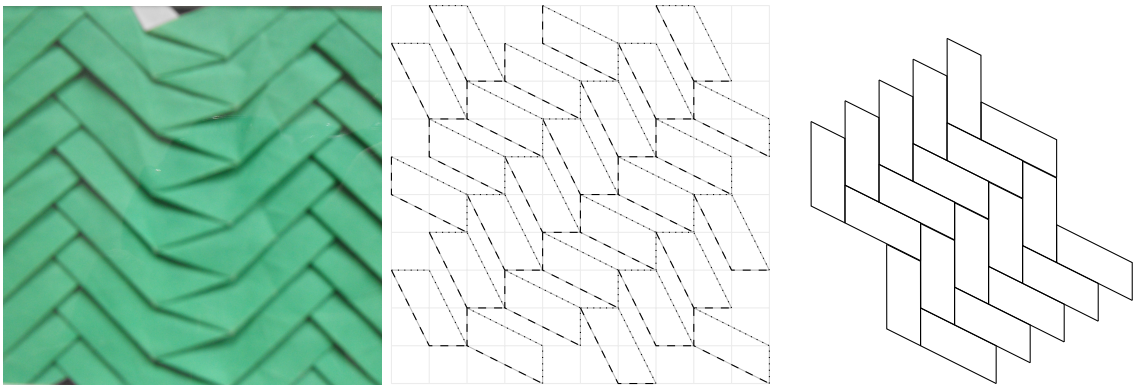


FIGURE 1. Herring bone approximation from a hinged parallellogram tiling

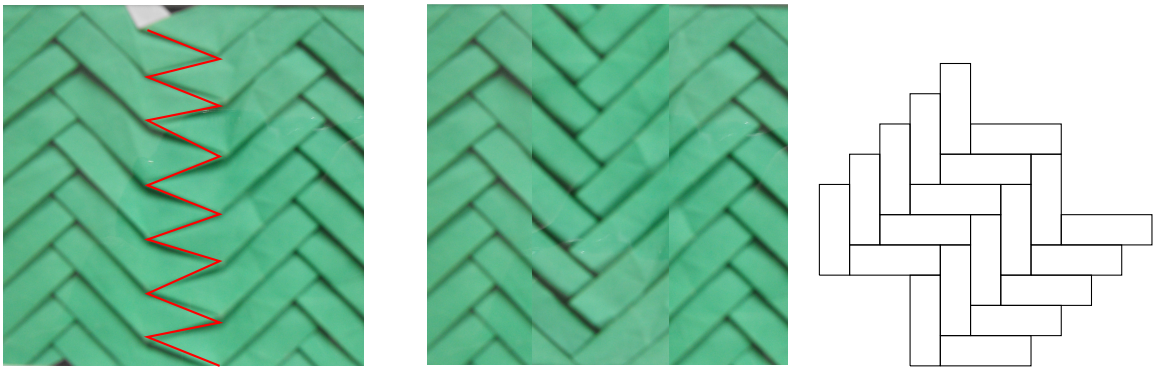


FIGURE 2. folds not part of a herring bone pattern, and how herringbone should look

folded back (red) and front (black) together:

selection of bricks from front and back:

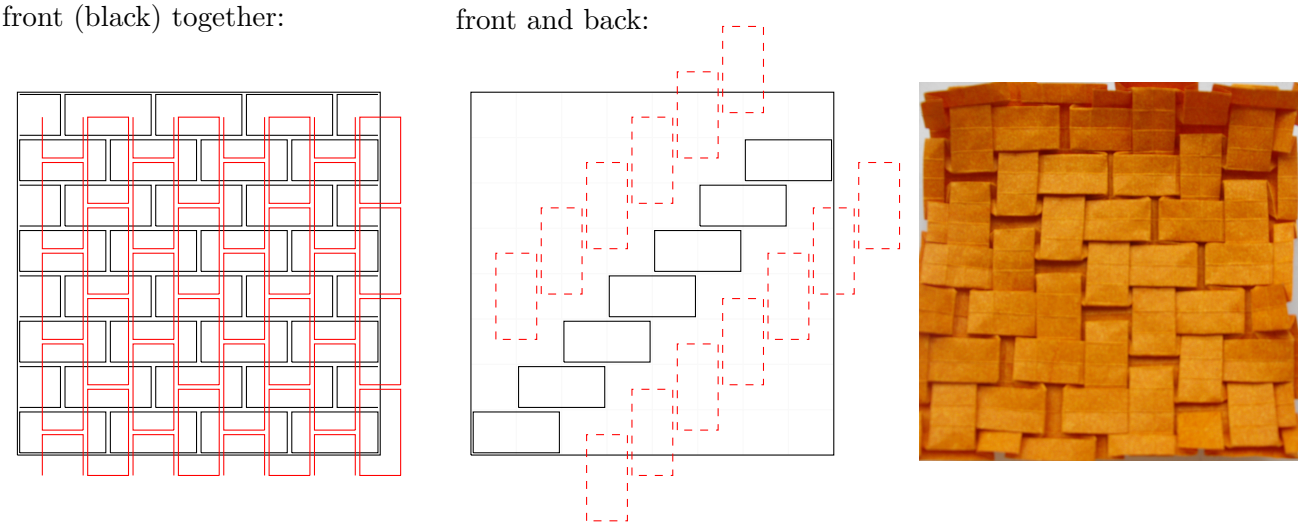


FIGURE 3. Trying to take bricks from the brick wall origami tessellation to make a herring bone tessellation

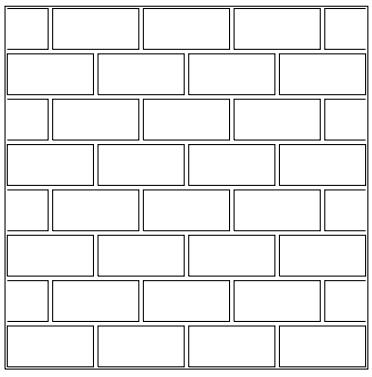
0.4. **Definition.** Every mathematician knows that you have to define a question first in order to be able to answer it. What do we mean by "herring bone pattern?" This is the kind of pattern found in knitting or in wooden flooring. An Example is given on the right in Figure 2

This is a tiling, all tiles have an identical quadrilatteral shape. typically, in fact, rectangular, non square, though, as in the example on the right in Figure 1, this does not have to be the case. The tiling is rotationally symmetric through 180° about the centre of each tile. Each tile is adjacent to 6 other tiles. 2 along each long edge, one along each short edge. The tiles adjacent to a given tile along the short edge are perpenciular to the given tile. We could also require that when rendered in origami, the ends of the tiles (in the longer direction) tuck under the perpendicular adjacent tiles at these ends.

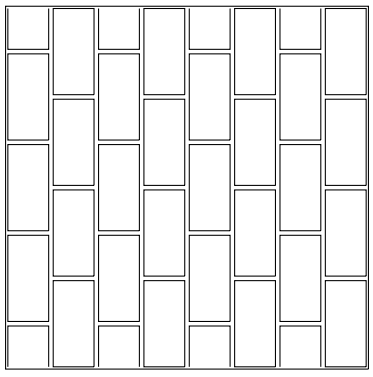
0.5. **false starts.** I took a couple of different origami tessellations as starting points, including the one in Figure 4 above – a little sink folding improves it a bit, but it's still not right. I tried another few ideas on the way, amongst a lot of sketches of other things that worked even less well as herringbone patterns! Though perhaps interesting if you don't want a herring bone!

0.6. **best approach so far.** Yoshihide Momotani's origami brick wall pattern [Momotani] looks like it might be a promising starting point:

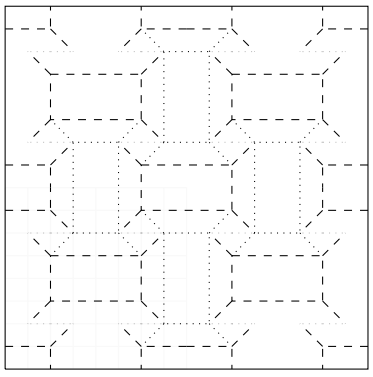
folded view:



folded view on back (more or less):



crease pattern (most of it):



Note, I am telling a lie in the middle picture – the back needs a shift, but I'd need to redraw rather than just rotate – see superimposed picture below where back is in red, to see what I mean.

If only we could have some of the bricks from the back move to the front, we would have something closer to a herring bone pattern, as in Figure 3

Well, it is possiblke to reverse some folds and bring some of the bricks on the back to be bricks on the front, at right angles to the intial bricks, e.g., as in the picture on the right in Figure 3... but although this can lead to all

some brick creases from back and front

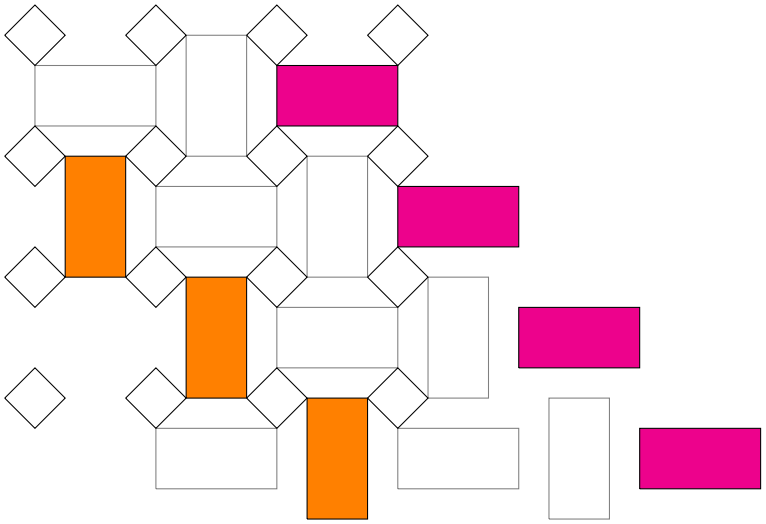


FIGURE 4. "bricks" from origami brick wall, to be incorporated in herring bone pattern

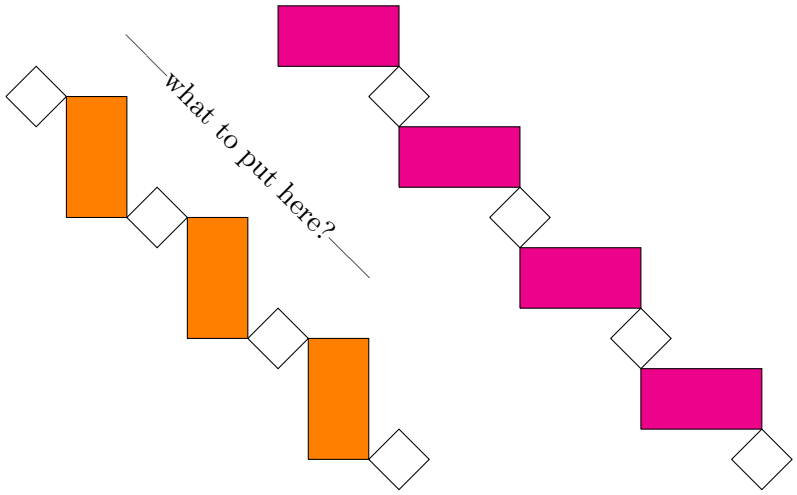


FIGURE 5. folds we might want to include

kinds of nice patterns — (which might lead us to ask... just how many different patterns can you actually get?! Just from reversing some folds, I mean. How to formulate that question properly and enumerate exactly? Just for fun!) — I can't manage to obtain a proper herringbone pattern.

Lets look at what we want, extracted from the crease pattern – diagonals of bricks, which I'll colour pink and orange, in two possible directions, as in Figure 4

Maybe we can just get rid of the bricks between the orange and pink ones, as in Figure 5

How can we fill in the space between these with some kinds of creases to bring together the bricks in the way we want? Well, let's bear in mind the 180° rule, and just fill in somehow or other, try different things, and see if any of them get us closer to what we want!

I couldn't get anything ulta-simple with them in the initial relative positions, so moved these sets of rectangles a bit closer, and added the green squares in between, as in Figure 6. This does fold flat, but everything is too much on top of each other, the tiles can't all be on the right side, etc, as in the photo on the right.

I want to keep the pink and orange bricks, so they appear as in the herringbone patern, but we don't want the other bricks to show... perhaps we can just distort those bricks a bit, as in Figure 7

Now, the folded origami is quite promissing, but the bricks are still too far apart - but it's easy to move them closer, by contracting the pale green rectangles. We make these thinner and thinner until they vanish all togerher, and we get Figure 8

Now, the folded origami theoretically resembles a herring bone pattern, but because in reality the paper does not fold quite together, the bricks are still too far apart. It would be better to have ends tucked under, as in

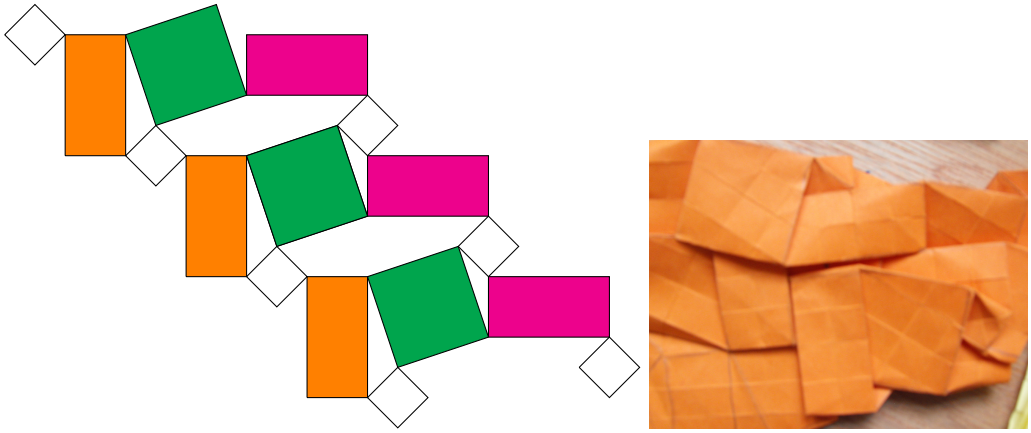


FIGURE 6. possible filling in

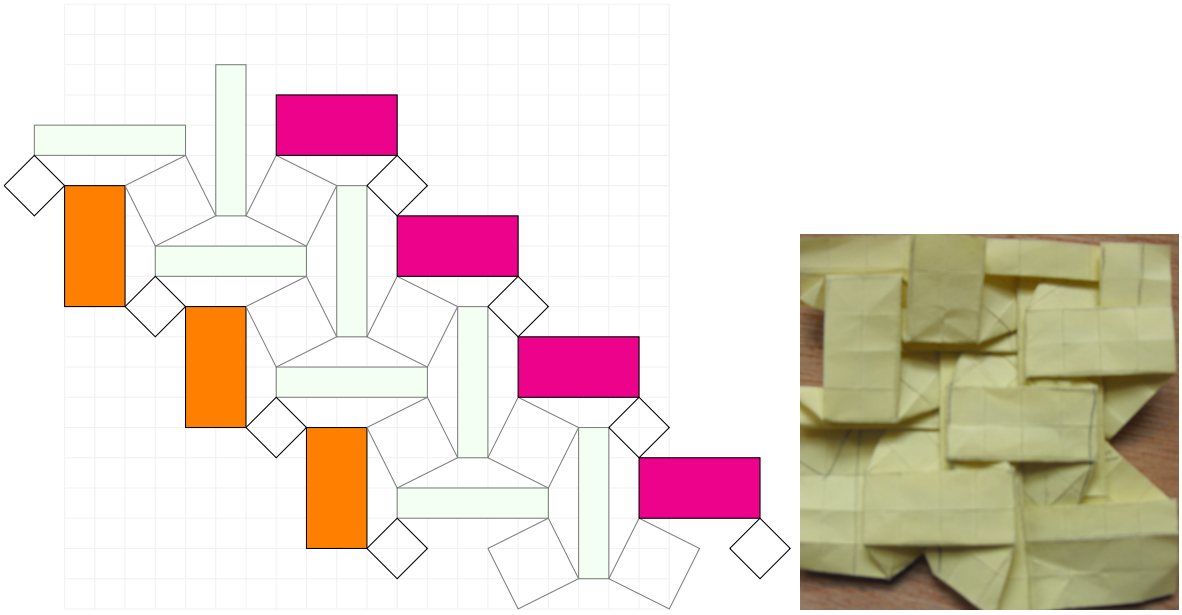


FIGURE 7. distorted brick wall crease pattern

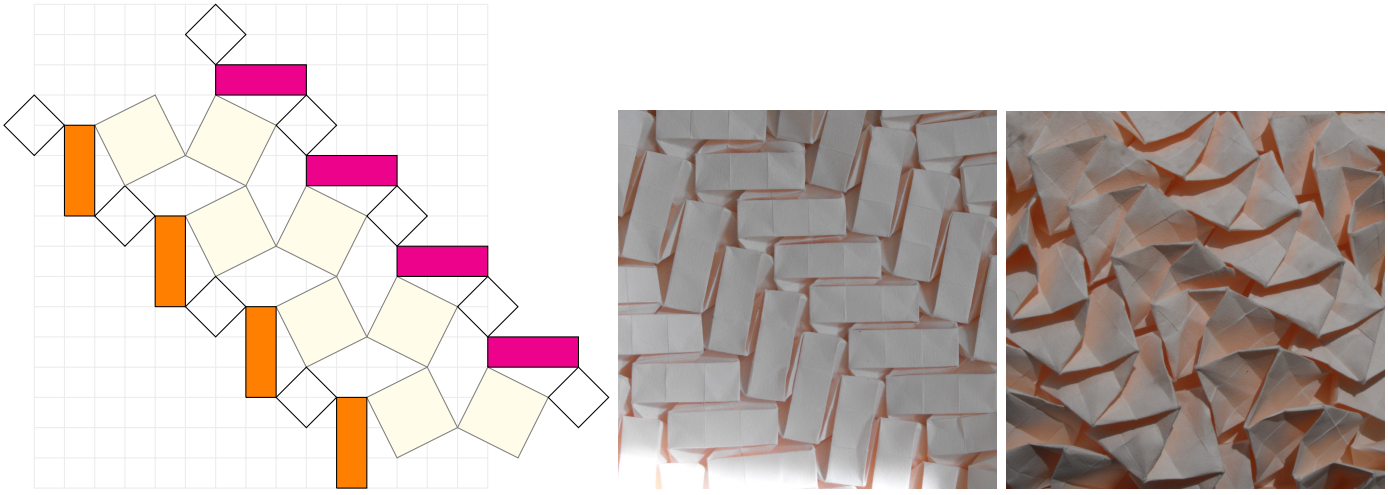


FIGURE 8. Even more distorted brick wall crease pattern... becomes herring bone... Back is on right

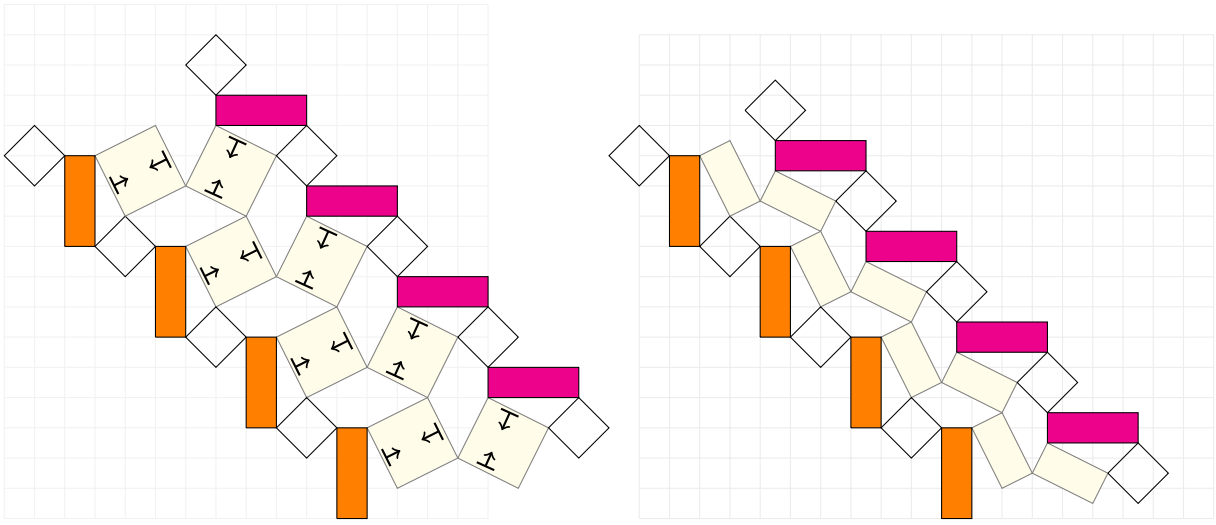


FIGURE 9. How to contract distorted brick wall crease pattern

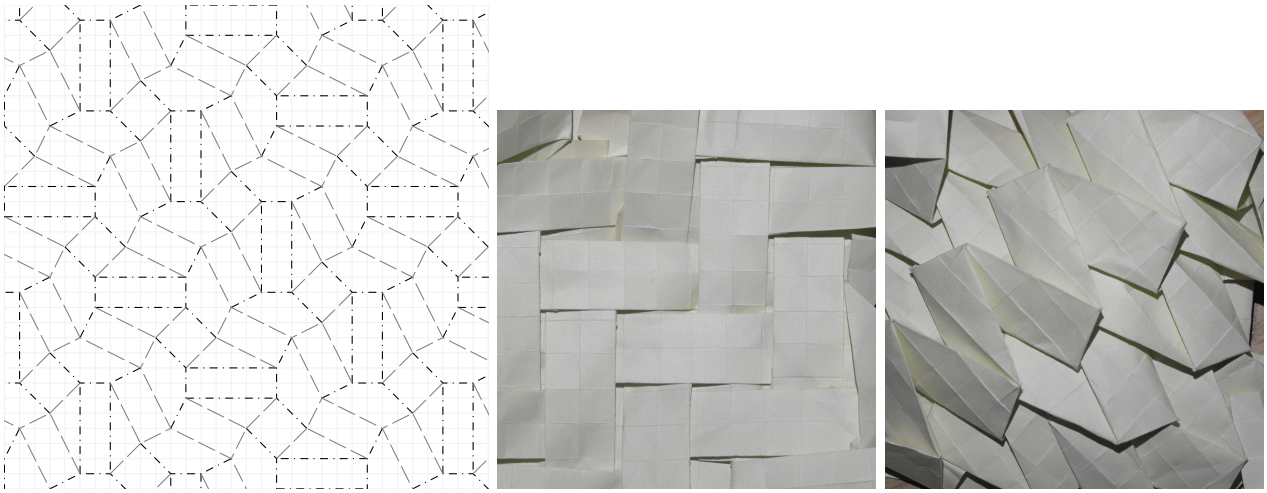


FIGURE 10. Crease pattern and front and back of a herring bone origami fold

some of the bricks in Figure 3. To achieve this, we can just move the bricks closer, by contracting the pale yellow squares in the way indicated by the arrows in Figure 9

The only problem with the result is that it does not actually fold flat, unless you make some little extra creases to tuck bits under (or you could even cut some slits for bits to go through, but I choose to just tuck bits under). The tucking under is a bit fiddly, and I don't want to put the extra creases on the crease diagram, since I don't think it would he helpful to fold them in advance. If you fold this, you will work out where the bits to be tucked are. Resulting fold is as in Figure 10

Note, in the right crease pattern of Figure 9 the bricks are a bit too close, since there is this tucking in problem. On the other hand, in Figure 8 they were not close enough to tuck under at all. There is a theorem that first year maths students learn - the intermediate value thereom, which leads us to hope that there will be a "just right" amount to contact, so the bricks are close enough, but no tucking under. But... I don't really want irrational coordinates... One could increase the divsions of the grid... if there's a way to do it, it can be done rationally...but... enough is enough for now...

I felt satisfied enough to stick with the result in Figure 10 for the moment.

There are lots of ways to vary this, (i.e., infinitely many, since this can be varied continuously) e.g., in Figure 11 there is another a crease pattern for a fold where the tiles come together, but do not tuck under. I only folded a small fragment of this pattern, shown next to the crease pattern.

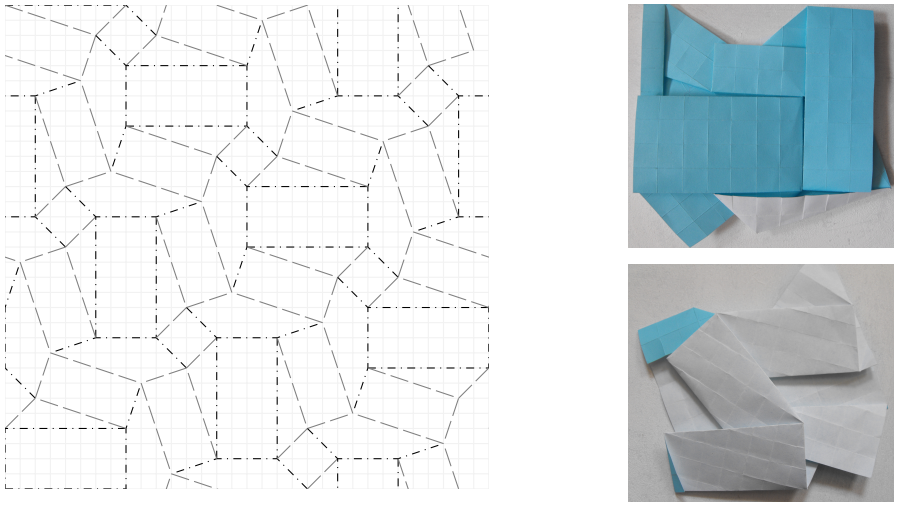


FIGURE 11. variation on origami in Figure 10

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