VARIATIONS ON ILAN'S CUBE TESSELLATION

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When I attended the Birminham BOS mini meeting [B] on 24th February 2019, Edward Holmes showed me a cube origami tessellation due to Ilan Garibi[I].

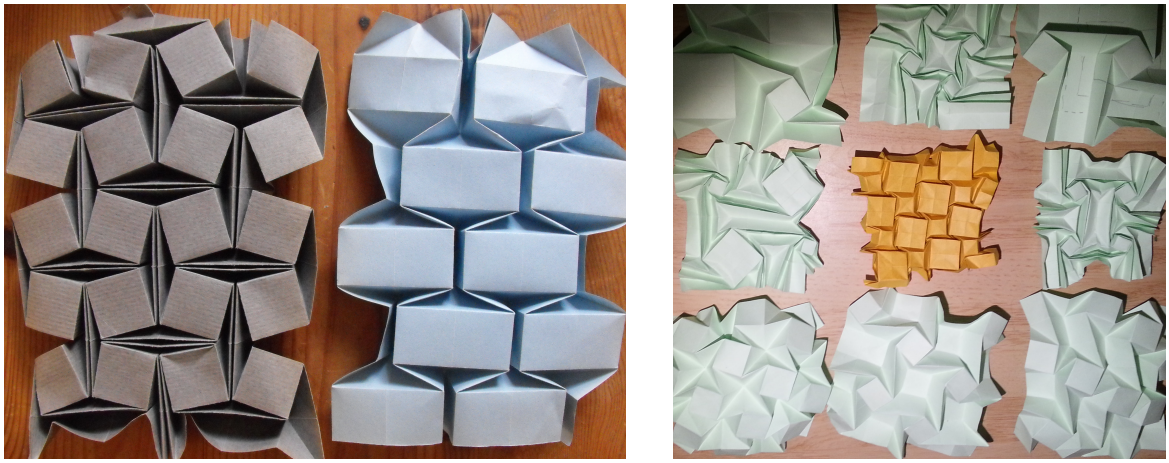

FIGURE 1. Left: Cube tessellation folded by Gary Kazin; Right: Variations on Ilans cube tessellation

After seeing Edward's folding of Ilan's cube tessellation I went home and folded the variations in Figure 1 right. I'll describe how I came to these.

Actually, I had seen the origami Edward showed me before, since Gary Kazin had sent me one in December 1997 (left in Figure 1), but I had not thought to look into it then, since generally I've just been doing flat tessellations. Anyway, I dug out the box with Gary's tessellations in, and started to think about them a bit.

What I really want to do here is investigate the relatioships between a few different cube tessellations and how to vary between them.

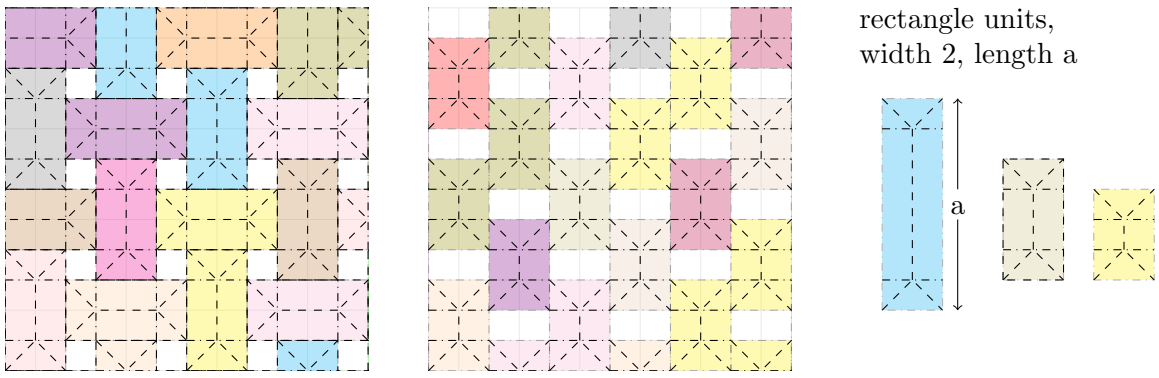

FIGURE 2. Crease patters for origamis in Figure 1 left

I first wanted to think of Gary's folds as ways of putting together the units on the right in Figure 2.

However, the crease pattern on the left in Figure 2 can also be thought of in terms of the cubes, as in the crease pattern left in Figure 10... which might in a way be more natural than thinking about the parts that get squashed flat and can't really be seen. Both view points can be further developed.

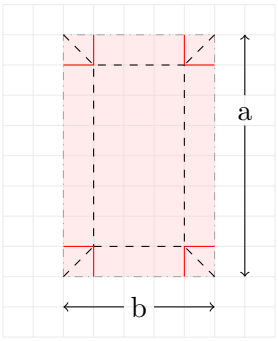

FIGURE 3. Rectangle unit expanding on unit in Figure 2

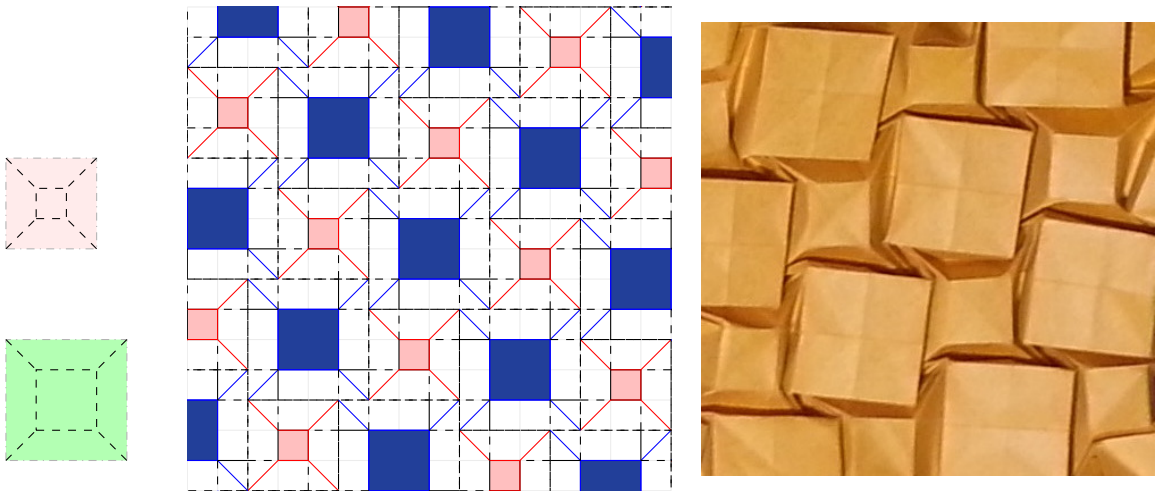

FIGURE 4. Two size square tessellation

First, how else could these kind of rectangles in Figure 2 be put together? First let's expand these rectangles, and vary the width as well as the length. Then we have a crease unit as in Figure 3. At each corner, there are two crease lines drawn in red, without direction shown, since for these two, at least one, but not necessarily both of the pair needs to be folded, and the crease directions are mountain if only one is folded, but could be a mountain valley pair if both are folded.

An example of putting together units like this is in Figure 4, which shows a tessellation of two different sized squares put together.

we can put together squares of size n and $n+1$ for any $n \geq 0$ (I mean n is the side length of the smaller square in the middle which can be seen when folded) in a similar manner. E.g., $n = 3$ in Figure 5, and with $n = 0$ in Figure 8 left photos. Left shows crease pattern. front photo is above photo of back. Right photos show cubes before the square of dimension 0×0 is folded, We can put together any kinds of rectangles with this kind of "dog leg" join; the basic "corner" unit is as in Figure 8. We can put this together in different ways, e.g., to make the the concertina like figures in the right of Figure 8. Figure 7 shows an example of what an origami might look like with this unit.

We can also try putting together just one size square or rectangle of the type in Figure 3; the case on the left in Figure 9 is very tight, and not so easthetically pleasant. Shifting columns of the unit makes a better tessellation, as on the right of Figure 9. The case with one size unit could also be improved by changing crease directions in alternate squares, so the lines in the pink areas are all reversed. A fragment is shown in lower left of Figure 9. It works out better if we use squares with size 2.

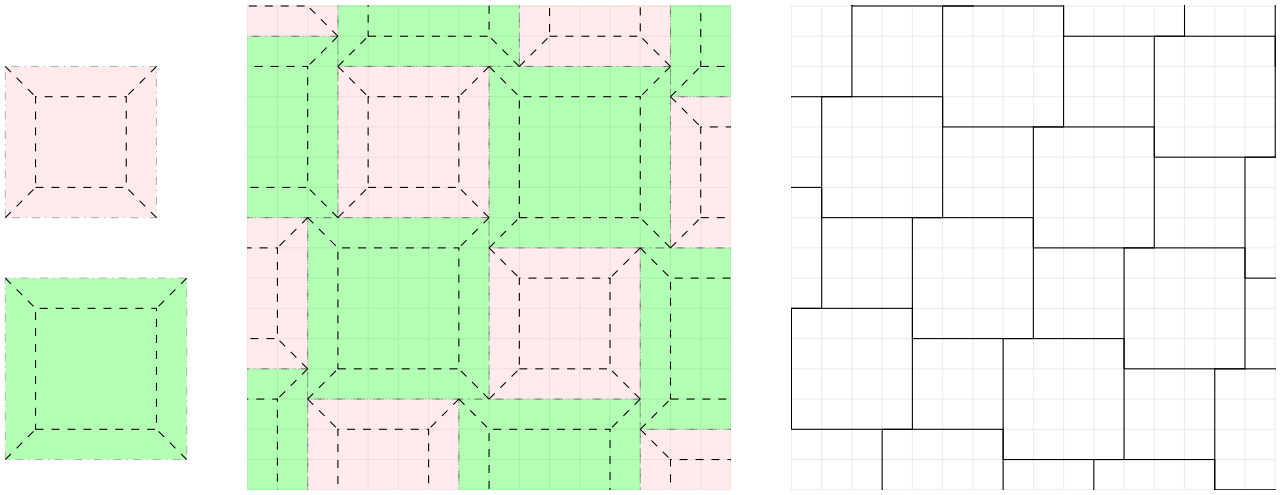

FIGURE 5. Crease pattern for square tessellation, $n = 3$; similarly we can put together any squares size n and $n + 1$. Right shows diagram of what it would look like folded, more or less. Some horizontal and vertical lines missing from crease diagram.

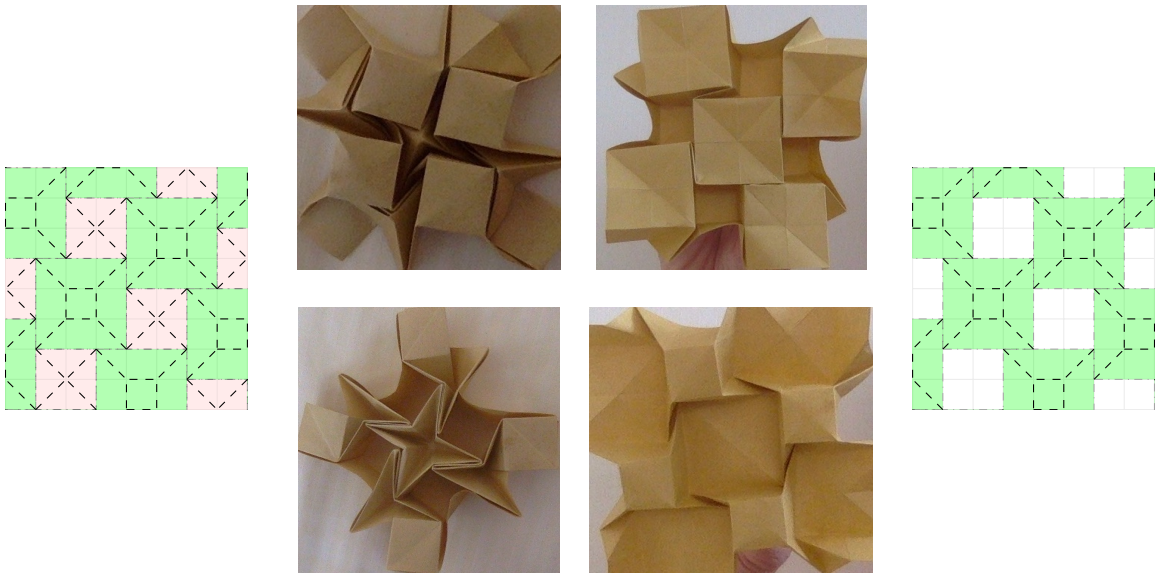

FIGURE 6. Two size squares, in the same sequence as in Figures 4, and 5, this time with $n = 0$; in right figure, the size 0 square units (the pink ones, with dot in the middle, size 0 square) are not collapsed (not folded). See Figures 3 and 4 for additional horizontal and vertical creases near the corner of each unit, which I've left out. All these photos are of the same piece of paper.

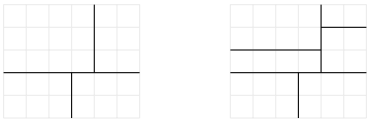

FIGURE 7. An origami that could be made from unit left in Figure 8. Picture on left shows allowed junction type at corners of rectangles with this unit

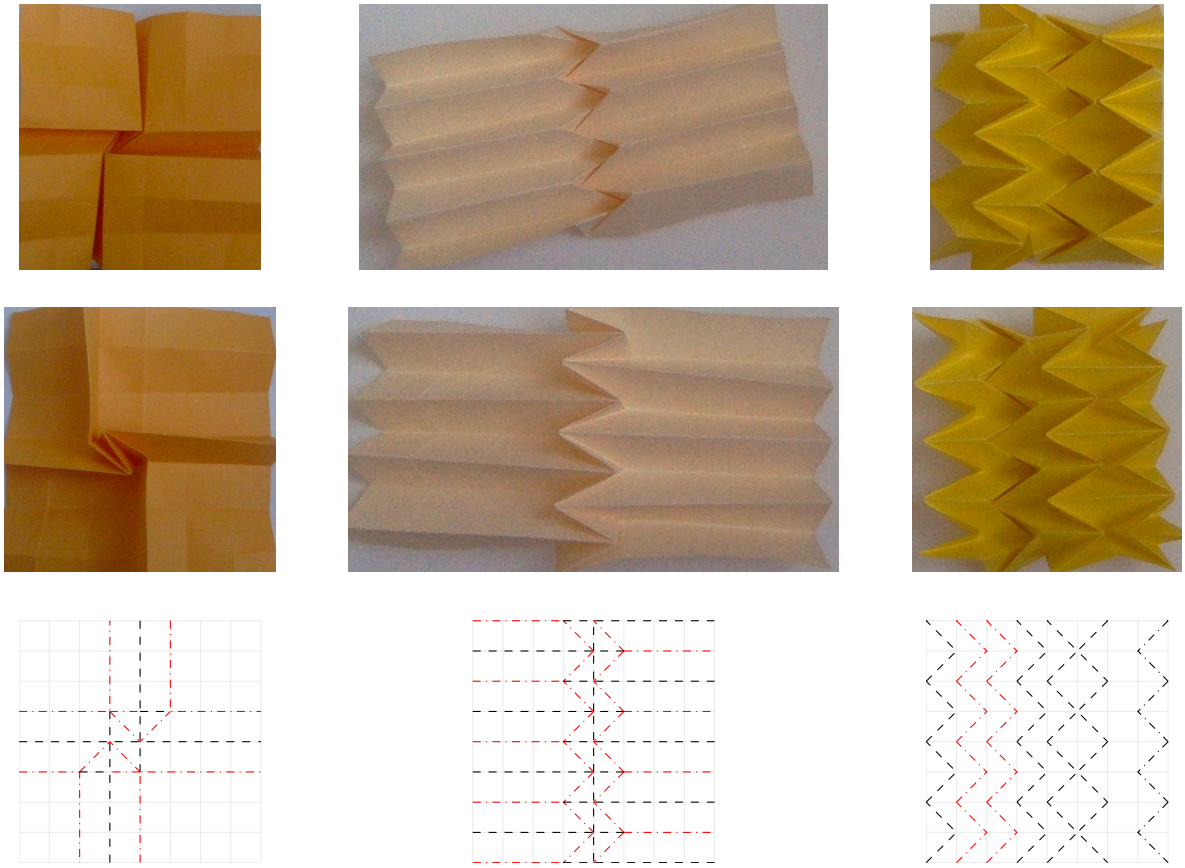

FIGURE 8. Left shows crease pattern unit extracted from cube tessellations in Figures 5 and 6. This is put together in a couple of different ways in the middle and right. Some crease lines missing in right diagram

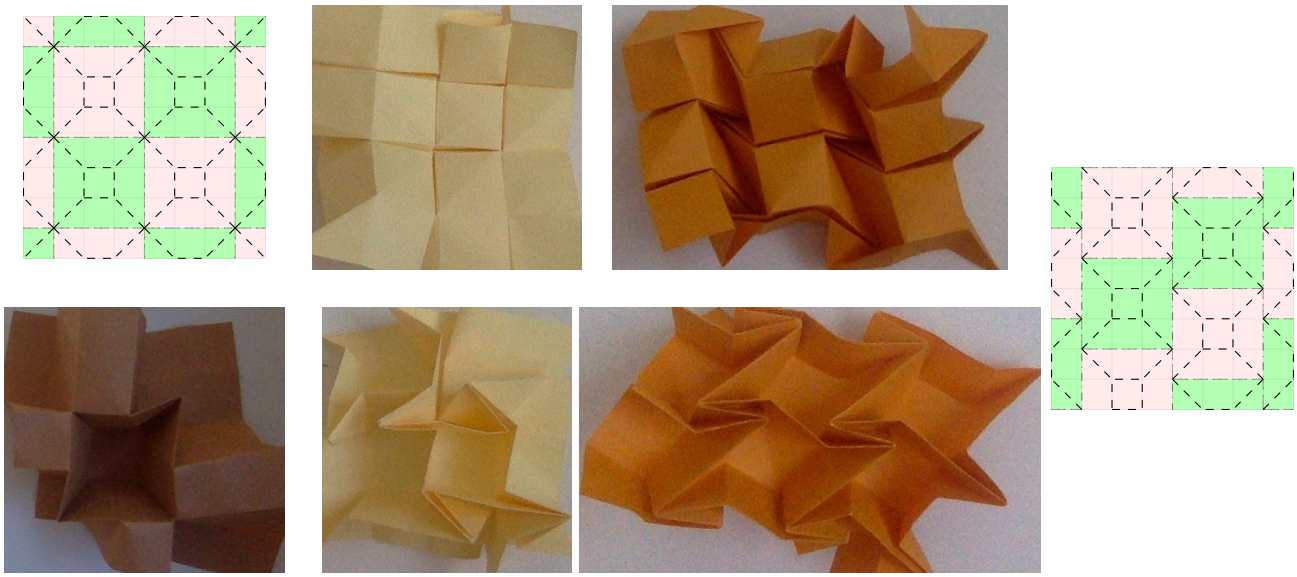

FIGURE 9. Just one size square as in Figure 3, put together in two or three ways

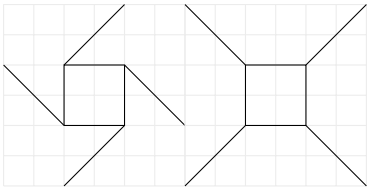

FIGURE 10. two kinds of cube for origami cube tessellation

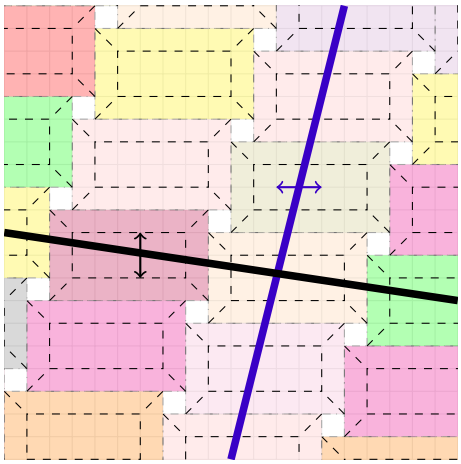

FIGURE 11. Element of a family of tessellations of rectangle unit in Figure 3. The bold lines show parts which can be expanded or contracted along, to change the dimensions of these rectangles. If you "cut" along these lines, you can move the paper further apart or closer together in the direction indicated, then glue back together. Similarly for parallel lines not drawn. This is an easy way to get different size rectangles in the same tessellation

0.1. **Two kinds of cubes.** We have two kinds of "cubes" in these origamis— some of the "cubes" pointed up, and have folds as in Figure 10 left, and some pointed down, and have folds as in Figure 10 right. Mostly we've looked at origamis with the cube on the right, though the very first picture in this paper uses the cube on the left. In this section we look at both kinds.

I was interested in putting these two units together in different ways; in investigating different positionings of the cubes relative to each other; in putting different sized cubes and rectangles next to each other; and in families of origami cube tessellations, and how they relate to each other.

In Figure 14, in the crease patterns, the squares that come "up" in the origami are coloured pink, and the squares or rectangles the go "down" are coloured dark green. I put the area that becomes the sides of the "down" squares in light green. From the first row to the second, the dark green squares have all been shrunk in one direction to become lines. From here to the third row, they have been shrunk in the other direction to become points. Edward Holmes informed me that this fold is due to Beth Johnson, or at least used by her in her sheep [J].

Here we see that these three are all just different members of the same infinite family of origami tessellations, just with a variation of parameter. Figure 11 trys to show the general case of the family. This pattern is built from units as in Figure 3, which are put together in such a way to leave a space for a cube, which looks like the left diagram in Figure 10.

In fact, these first three origami tessellations can all be combined into one, as shown in Figure 12 and 13. The same method can be used to put any two arbitrary members of this pattern together into one piece of paper, bridged by a third member, and in fact many members could be combined in the same sheet.

In the lower row of Figure 14, different sized rectangles and squares are put together in different ways... not yet very exhaustivly done, but gives a few of the many options. In the last case, the dark green rectangles have been collapsed, but red squares have been encorporated instead. In this case the small pink square rotates one more quarter turn. I guess it should be possible to include it in tessellations where it rotates arbitrarily much.

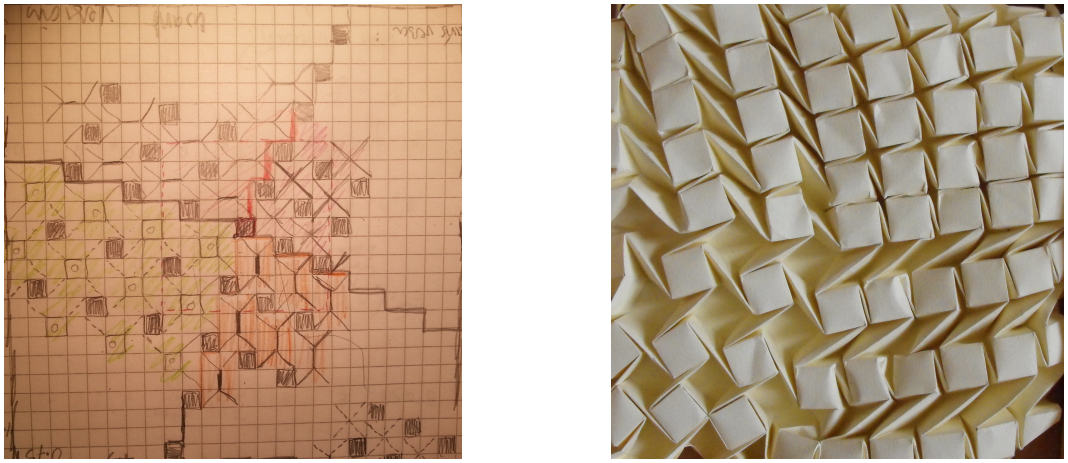

FIGURE 12. transition between cube tessellations - four tessellations in one sheet

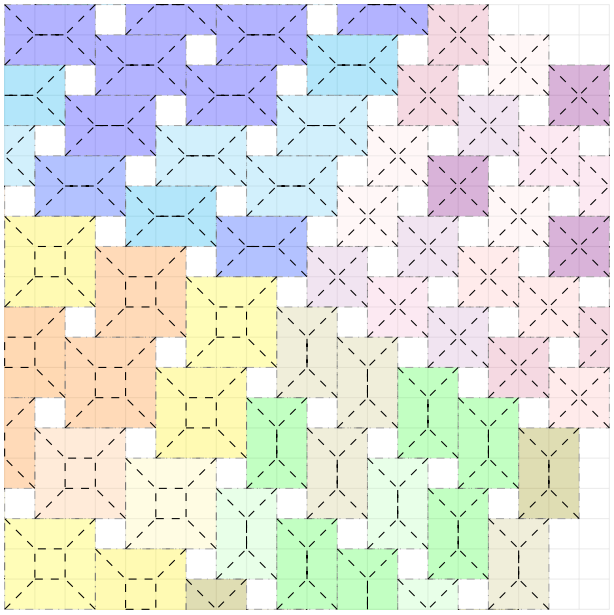

FIGURE 13. transition between cube tessellations - crease pattern for origami in Figure 12

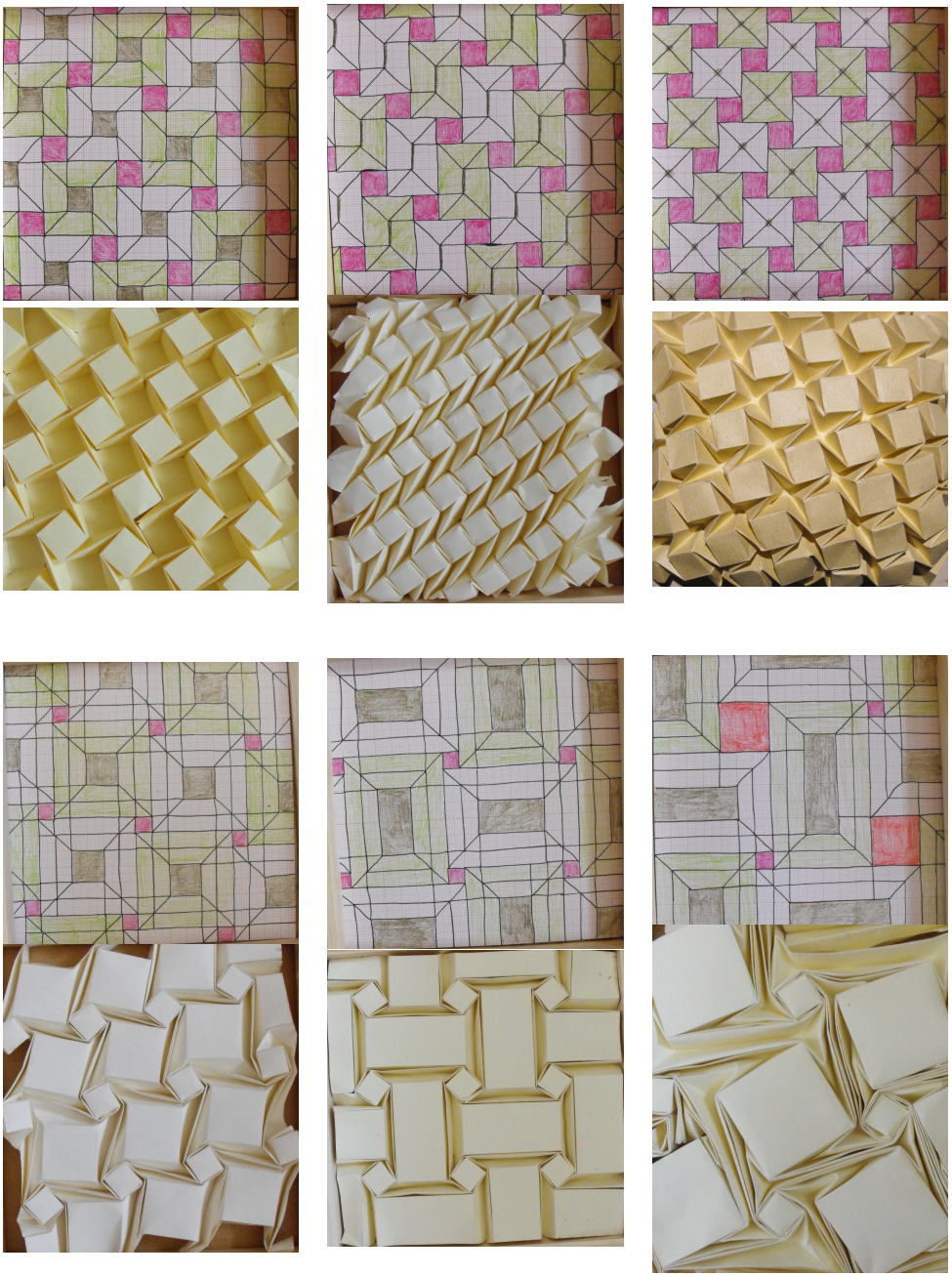

FIGURE 14. Variations on Ilan's cube origami

REFERENCES

[B] David Venables runs the Birmingham British Origami Society (BOS) mini meetings. http://www.britishorigami.info/society/meetings/mini-meetings/

[I] Origami tessellations for everybody, by Ilan Garibi https://www.origami-shop.com/en/origami-tessellations-everyone-free-chapters-free-ebook-xml-218_2168_2169-11548.html?ref=2&affiliate_banner_id=9

[J] Beth Johnson's Sheep model http://bethjohnsonorigami.com